ID # 7510

Section # C

Subject # Differentional Equat

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Question #1

① The order of matrix A is $m \times p$ and the order of B is $p \times n$ then the order of matrix AB is?

Ans) $A_{m \times p}$ and $B_{p \times n}$

then $AB = m \times n$

The order of AB is $m \times n$.

② The number of non-zero rows in an Echelon form?

Ans) The number of non-zero rows in Echelon form is called Rank.

Example:-

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Its row Rank is three.

(3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matri-

then $a = ?$

~~Sol: The number~~

Sol:- We know that for Singular matrix $|B| = 0$

So $|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$

$= 1 \times a - 2 \times 4 = 0$

$1 \times a - 2 \times 4 = 0$

$a - 8 = 0$

$\boxed{a = 8}$ Ans

(4) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ the $|A| = ?$

Sol:- $|A| = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

taking modulus of a matrix A.

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$|A| = 2i \times (-i) - i \times i$

$|A| = -2i^2 - i^2$ $\quad (i^2 = -1)$

$|A| = -2(-1) - (-1)$

$|A| = 2 + 1$

$\boxed{|A| = 3}$ Ans

(5) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

Sol:- $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

Here in matrix A diagonal elements is same i.e 9. So it is Scaler Matrix.

(6) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol: $\frac{dy}{dx} + 2xy = y$

$$\Rightarrow \frac{dy}{dx} = y - 2xy$$

$$\Rightarrow \frac{dy}{dx} \cancel{=} \frac{y(1-2x)}{1}$$

$$\Rightarrow dy = y(1-2x)\,dx$$

$$\Rightarrow dy = y(1-2x)\,dx$$

integrate

$\Rightarrow \int \frac{dy}{y} = \int (1-2x)\, d$

$\Rightarrow \ln y = \int 1\, dx - 2\int x\, d$

$\Rightarrow \ln y = x - \frac{2x^2}{x} + c$

$\Rightarrow \boxed{\ln y = x - x^2 + c}$ Ans

⑦ The order and degree of different equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is = ?

Ans) Order = 1

Degree = 3

⑧ The order and degree of differential $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is = ?

Ans) Order = 2

Degree = 1

a) The differential equation $2\frac{dy}{dx} + x^2y = 2x+3, y(0)=$

Sol:

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2}x^2y = 2x+3 \quad y(0)=5$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2}x^2y = x + \frac{3}{2} \quad \div \text{ by } 2$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2}x^2y = x + \frac{3}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}x^2y + \frac{1}{2}x + \frac{3}{2}$$

Integrate

$$\Rightarrow y = -\frac{1}{2}\frac{x^3}{3}y + \frac{1}{2}\frac{x^2}{2} + \frac{3}{2}x + C_1$$

$$\Rightarrow y = -\frac{1}{6}x^3y + \frac{1}{4}x^2 + \frac{3}{2}x + C_1 \longrightarrow (*)$$

How use Condition

at $x=0$, $y=5$. Put in above eq

$5 = 0+0+0+C_1$

$C_1 = 5$

$$\boxed{y = \frac{1}{6}x^3y + \frac{1}{4}x^2 + \frac{3}{2}x + 5}$$

Required Particular Solution

(10) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is = ?

Sol:- Expand w.r.t to Column number

$\Rightarrow 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$

$\Rightarrow 1\cdot(bc - cb^2) - 1(ac^2 - ac) + 1(ab^2 - a^2b)$

$\Rightarrow (bc^2 - cb^2) - (ac^2 - ac) + (ab^2 - a^2b)$

$\Rightarrow bc(c-b) - ac(c-a) - ab(b-a)$

$\Rightarrow bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b$

$\Rightarrow (c-b)\{bc - ac - ab - a^2\}$ Ans

Question #2

(1) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ as the product

of factors which are linear in a, b, c.

Sol:- ~~Taken~~

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Taking Common:- ~~from b~~

$$= abc \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

expanding by R_1

$$= abc\left[1\begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - 1\begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + 1\begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} \right]$$

$$= abc\left[bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b \right]$$

$$= abc\left[bc^2 - ac^2 - b^2c + a^2c + ab^2 - a^2b \right]$$

$= abc\left[bc^2 - ac^2 - b^2c + a^2c + ab^2 - a^2b\right]$

$= abc\left[c^2(b-a) - c(b^2-a^2) + ab(b-\right.$

$= abc\left[c^2(b-a) - c(b+a)(b-a) + \right.$

$= abc(b-a)\left[c^2 - c(b+a) + ab\right]$

$= abc(b-a)\left[c^2 - cb - ca + ab\right]$

$= abc(b-a)\left[c(c-b) - a(c-b)\right]$

$= abc(b-a)(c-b)(c-a)$ Ans

Question #2 Part # B

Find the Eigen Value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

For Eigen Value Consider $A-\lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$

$|A - \lambda I| = 0$

$$\Rightarrow \left| \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-2 & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} \right| = 0$$

Using $R_3 - R_2$

$-1-3+\lambda$

$3-\lambda+1$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ 0 & -4+\lambda & 4-\lambda & 0 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by column first

$$(2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} + (-1)\begin{vmatrix} -1 & -1 & 0 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by C_3 Expand by C_3

$$\Rightarrow (2-\lambda)\left[-1\begin{vmatrix} 4+\lambda & 4-\lambda \\ -1 & -1 \end{vmatrix} + (2-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ -4+1 & 4-\lambda \end{vmatrix}\right]$$

$$\Rightarrow -1\left[(2-\lambda)\begin{vmatrix} -1 & -1 \\ -4+\lambda & 4-\lambda \end{vmatrix}\right] = 0$$

$$-(2-\lambda)(4-\lambda+4-\lambda)+(2-\lambda)^2((3-\lambda)(4-\lambda)$$
$$-4+\lambda) -1((2-\lambda)(\lambda-4-4+2)=0$$

$$\Rightarrow (\lambda-2)(8-2\lambda)+(\lambda-2)^2(\lambda^2-6\lambda+8)-(\lambda-2)$$
$$(8-2\lambda)=0$$

$$\Rightarrow (\lambda-2)^2(\lambda^2-6\lambda+8)=0$$

$$\Rightarrow (\lambda-2)^2=0 \quad \text{or} \quad \lambda^2-6\lambda+8=0$$

$$\Rightarrow \lambda=2,2 \quad \& \quad \lambda = \frac{-(-6)\pm\sqrt{(-6)^2-4(8)}}{2(1)}$$

$$\Rightarrow \lambda = \frac{6\pm\sqrt{36-32}}{2}$$

(12)

$$\Rightarrow \lambda = \frac{6 \pm 2}{2}$$

$$\Rightarrow \lambda = \frac{8}{2}, \frac{4}{2}$$

$$\Rightarrow \lambda = 4, 2.$$

Eigen Values are 2, 2, 2, 4

Question # 3

The rate of change in the form of differentional equation is given by.
$(x^2+3y^2)dx - 2xydy = 0$. Find the general Solution as $x=2$ and $y=6$

Sol. $(x^2+3y^2)dx = 2xy\,dy$; $y(2) = 6$

$= 2xydy = (x^2+3y^2)\,dx$

$= 2xy\,\frac{dy}{dx} = x^2+3y^2$

$= \frac{dy}{dx} = \frac{x^2+3y^2}{2xy} \longrightarrow ①$ homogenous

Put $y = Vx$

$$\frac{dy}{dx} = V + x\,\frac{dV}{dx}$$

$$V + x\,\frac{dV}{dx} = \frac{x^2+3V^2x^2}{2xVx}$$

$$\Rightarrow V + x\frac{dv}{dx} = \frac{\cancel{x}(1+3V^2)}{2\cancel{x}V}$$

$$\Rightarrow V + x\frac{dV}{dx} = \frac{1+3V^2}{2V}$$

$$eq\ (4) \Rightarrow \frac{x^2+y^2}{x^3} = 5$$

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y = \sqrt{5x^3 - x^2} \quad Ans$$