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Q No 1 ①

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2)$$

$y(0) = 0$ so $t = 0, y = 0.$

$dy = e^{y} \cdot e^{-t} \sec(y)(1+t^2)\, dt.$

$$\frac{1}{e^{y} \cdot \sec(y)} dy = (1+t^2)\, e^{-t}\, dt$$

as $\cos(y) = \frac{1}{\sec(y)}$

L.H.S R.H.S.

$$\int e^{-y} \cos y\, dy = \int (1+t^2)\, e^{-t}\, dt.$$

using integration by parts.

$$e^{-y} \int \cos y\, dy - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) =$$

$$(1+t^2) \int e^{-t} - \int \left(\int e^{-t} \cdot \frac{d}{dt} (1+t^2) \right) \rightarrow eq①$$

L.H.S.

$$e^{-y} \int \cos y\, dx - \int \left(\int \cos y \cdot d/dy\; e^{-y} \right).$$

$e^{-y}\sin y - \int(\sin y . e^{-y}(-1)$

$e^{-y}\sin y + \int(\sin y . e^{-y})$

$e^{-y}\sin y + \int(e^{-y}\sin y)$

Again using integration by parts.

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int \sin y \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(-\cos y \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int(\cos y e^{-y})$

Since $\int(\cos y e^{-y}) = L.H.S.$

Since it is again same to the first one so L.H.S will become

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S.$

$2\,LHS = e^{-y}(\sin y - \cos y)$

$LHS = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S.

$$\int (1+t^2)\, e^{-t}\, dt.$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \tfrac{d}{dt}(1+t^2)\right)$$

$$(1+t^2)\, e^{-t} - \int(-e^{-t}(2t))$$

$$-(1+t^2)\, e^{-t} + \int (2t)\, e^{-t})$$

Again using integration by parts..

$$-(1+t^2)\, e^{-t} + (2+\int e^{-t} - \int\left(\int e^{-t}\, \tfrac{d}{dt}\, 2t\right))$$

$$-(1+t^2)\, e^{-t} + (-2t\, e^{-t} - \int(-e^{-t}\, 2)$$

$$-(1+t^2)\, e^{-t} + (-2t\, e^{-t} + \int (2e^{-t}))$$

$$-(1+t^2)\, e^{-t} + (-2t\, e^{-t} - 2e^{-t}) + c.$$

$$-(1+t^2)\, e^{-t} - 2t\, e^{-t} - 2e^{-t} + c$$

$$-e^{-t} - e^{-t}\, t^2 - 2te^{-t} - 2\, e^{-t} + c$$

$$\Rightarrow -(t^2+2t+3)\, e^{-t} + c = R.H.S$$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2 + 2t + 3)e^{-t} + c$$

we know that

$t = 0$, $y = 0$. put it above

$$\frac{1}{2}(0-1) = -3 + c$$

$$c = 5/2$$

put value of c.

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2 + 2t + 3)e^{-t} + \frac{5}{2}$$

Ans

Q2 $\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)$

$dy=0.$

Sol:

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}\longrightarrow (1).$$

This is Homogeneous Differential eq in x and y to solve this put

$y=vx.$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}.$$

thus eq (1) become

$$v+x\frac{dv}{dx}=\frac{\sqrt{x+vx}+\sqrt{x-vx}}{\sqrt{x+vx}-\sqrt{x-vx}}$$

$$v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

(2)

$$\Rightarrow v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times$$

$$\frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}\left(1+\sqrt{1-v^2}\right)}{\cancel{2}v}$$

$$v + \frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

(3)

$$x \frac{dv}{dx} = \frac{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

Taking Integral on b.s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right.} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx.$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx.$$

(4)

$$\ln\left(1+\sqrt{1-v^2}\right) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = c_1 \qquad \because \frac{1}{c} = c_1$$

The Required Solution.

①

Q3 $(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$

Sol:

$f(D)y = f(x)$

As it is non-homogeneous linear equation so solution as.

$y = yc + yp \rightarrow ①$

Complementary solution yc.

$\cancel{D^4}$ $D^4 - D^2 = 0 \Rightarrow D^2(D^2+1)=0$

Either $D^2=0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \Rightarrow D^2=-1$

$D=\sqrt{-1} \Rightarrow \boxed{D=i}$ or $D=0+i$

Roots are zero and Complex

$yc = c_1e^{0x} + c_2 e^{0x}(c_2\cos x + c_3\sin x)$

$yc = c1 + c_2\cos x + c_3\sin x$

$yp = \frac{1}{f(D)} F(x)$

(2)

$$y_P = \frac{1}{D^4+D^2}\left(3x^2+4\sin x-2\cos x\right)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4+D^2.$

at $D=0 \Rightarrow f(D)=0$

So

$f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(D)=0$

again differentiating

$f''(D) = 12D+2$

So for $D=0$.

$f''(0) = 12(0)+2 = 2.$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

(3)

$\Rightarrow \quad y_P = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} 2\cos x$

putting $D=0$ in all

So

$$y_P = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_P = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So

putting in eq (1).

$$y = C_1 + C_2\cos x + C_3\cos x + \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x.$$

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4}$$