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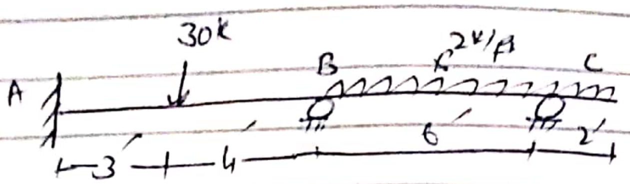
Structure analysis II

Civil Engineering

Date: 25/sep/2020

Q1

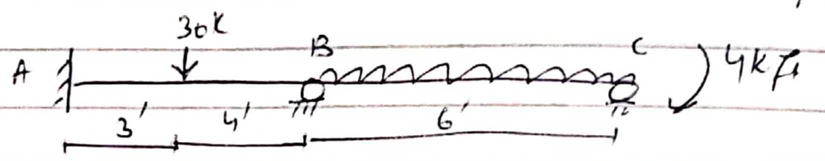


Sol

Step 01 Determining K.I ~~...~~

$= 5^0$

So we have to reduce the extended portion



$\because \frac{2 \times 2}{1} = 4k$

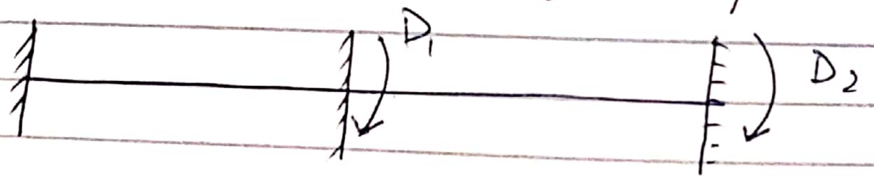
Now

~~...~~

$K.I = 2^0$

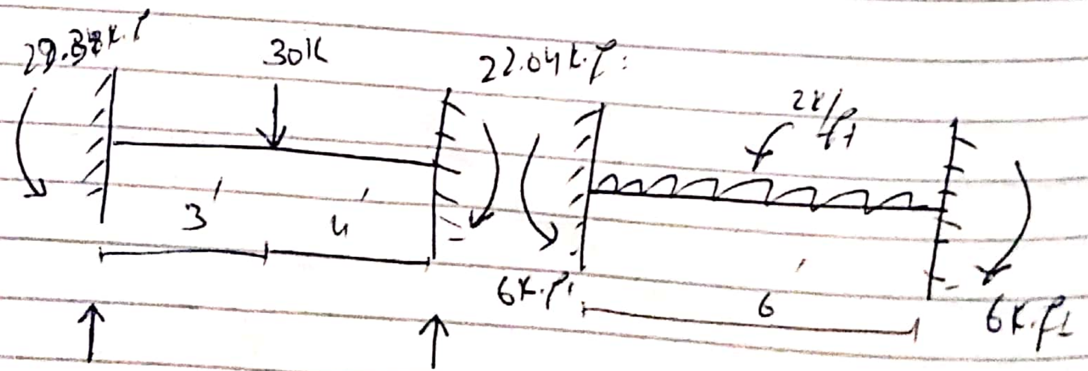
Step 02

Determine the unknown Joint Displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \qquad \begin{pmatrix} AD_1 \\ AD_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step #3 Compute [ADL] matrix



(2)

For load not on a center or mid

For left end $\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{7^2} = 29.38\ K.Ft$

For Right end $\frac{Pa^2b}{L^2} = \frac{(30)(3^2)(4)}{7^2} = 22.04\ K.Ft$

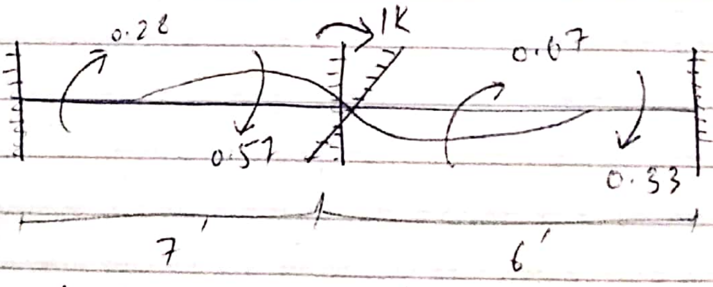
For UDL $\frac{wL^2}{12} = \frac{(2)(6)^2}{12} = 6\ K.Ft$

$ADL_1 = 22.04 - 6 = 16.04\ K.Ft$
$ADL_2 = 6\ K.Ft$

Step #4 compute (δ)

$$\delta = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

(a) $D_1 = 1K$ $D_2 = 0$



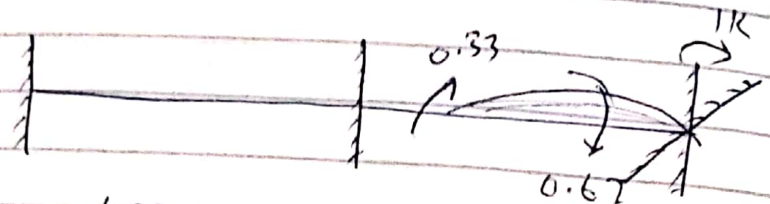
$\frac{4EI}{7} = 0.57$ | $\frac{2EI}{6} = 0.33$

$\frac{4EI}{6} = 0.67$ | $\frac{2EI}{7} = 0.28$

$\delta_{11} = 0.57 + 0.67$
$= 1.24$

$\delta_{21} = 0.33$

(b) $D_1 = 0 \quad D_2 = 1K$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$\delta_{12} = 0.33$$

$$\delta_{22} = 0.67$$

$$\delta = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step#5 Compute (D)

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}^{-1} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.9 \\ 3.89 \end{bmatrix}$$

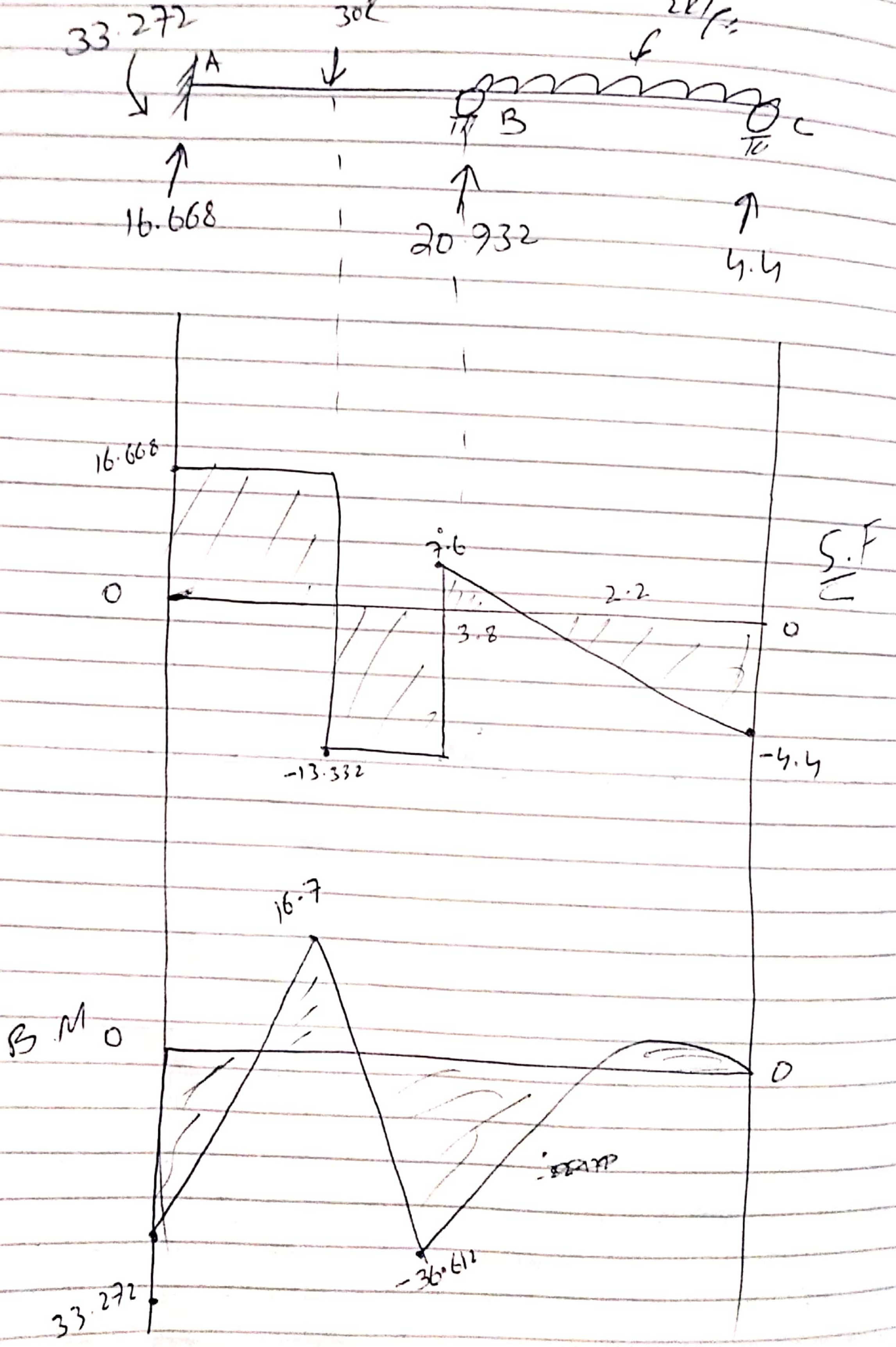
$$[AM] = [AML] + [AMD] \times [D]$$

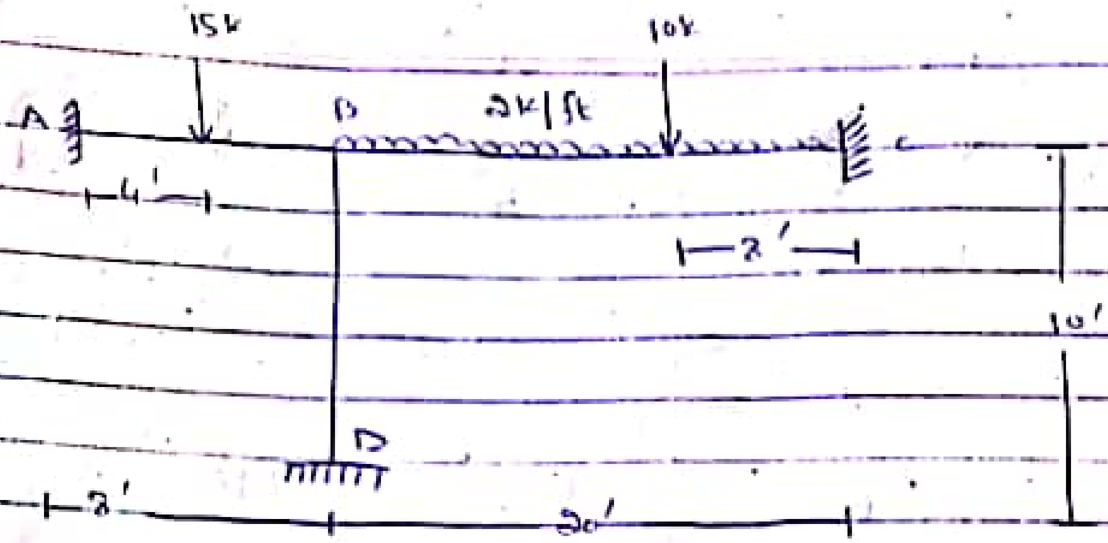
$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 6 \\ 6 \\ 29.38 \\ 22.04 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} -0.12 & 0 \\ 0.12 & 0 \\ -0.16 & -0.16 \\ 0.16 & 0.16 \\ -0.28 & 0 \\ 0.57 & 0 \\ -0.67 & -0.33 \\ -0.33 & 0.67 \end{bmatrix} \times \begin{bmatrix} -13.9 \\ 3.89 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ 6 \\ 6 \\ 29.38 \\ 22.04 \\ 6 \\ 6 \end{bmatrix} + \begin{bmatrix} +1.668 \\ -1.668 \\ +1.6016 \\ -1.6016 \\ +3.892 \\ -7.9 \\ +8.0293 \\ -6.0807 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \begin{bmatrix} 16.668 \\ 13.332 \\ 7.6 \\ 4.3984 \\ 33.272 \\ 14.14 \\ 14.02 \\ 10 \end{bmatrix}$$

(5)


33.272
30k
2k/ft
A
B
C
16.668
20.932
4.4
16.668
0
7.6
2.2
3.8
0
-13.332
-4.4
S.F
16.7
B.M
0
0
-36.612
33.272



Sol

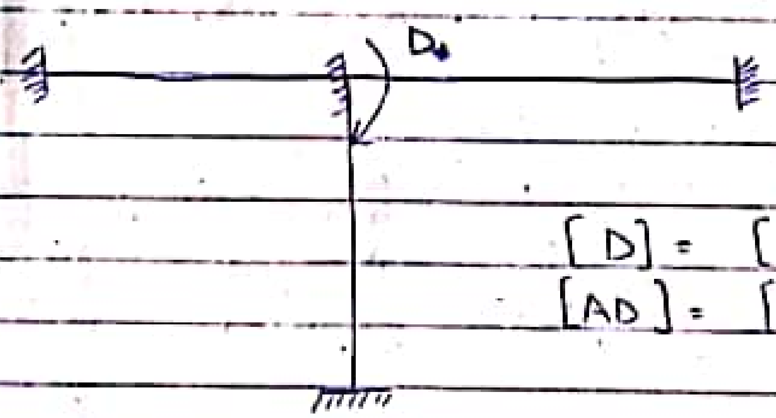
Step #1.-

Determine Kinematic Indeterminancy.

K.I = 1°

Step #2 :-

Determine Unknown Joint Displacement.

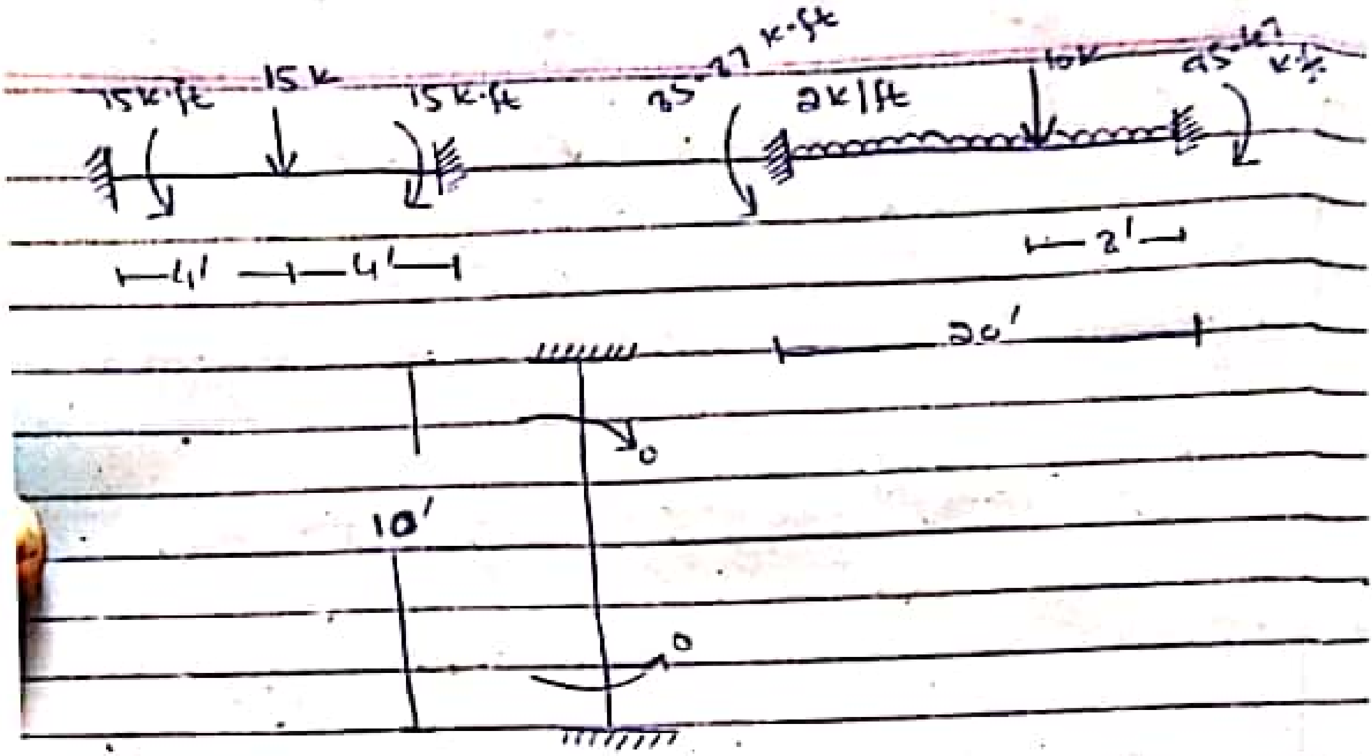


[D] = [?]

[AD] = [0]

Step #3 :-

Compute [ADL] Matrix.



=> Point Load at Center :-

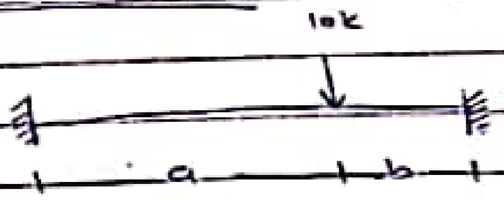
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip·ft}$$

=> Uniformly Distributed Load :-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k·ft}$$

=> Point Load (Not at mid) :-

Suppose :-



For Left End :-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k·ft}$$

For ~~Left~~ Right End :-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k·ft}$$

So Total moment at left end:-

$19.2 + 66.67 = 85.87 \text{ k-ft}$

Similarly at right End:-

$28.8 + 66.67 = 95.47 \text{ k-ft}$

So $[ADL] = -85.87 + 15 = -70.87 \text{ k-ft}$

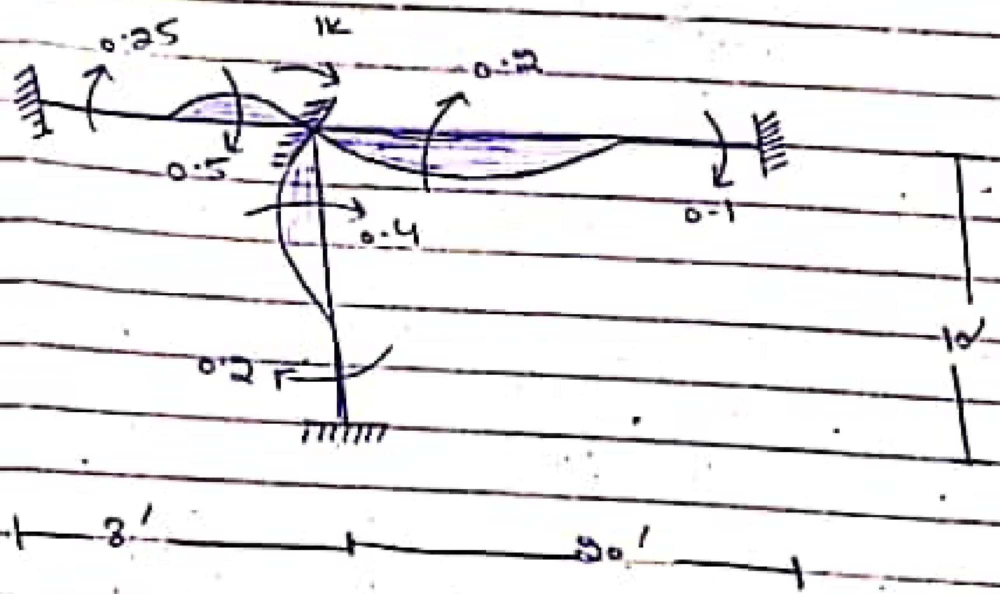
Step #4:

Determine $[S]$ matrix

$$[S] = \begin{bmatrix} S_{11} \end{bmatrix}$$

Now,

$D = 1k$



$\Rightarrow \dfrac{4EI}{8} = 0.5 \qquad \dfrac{2EI}{8} = 0.25$

$\Rightarrow \dfrac{4EI}{20} = 0.2 \qquad \dfrac{2EI}{20} = 0.1$

$\Rightarrow \dfrac{4EI}{10} = 0.4 \qquad \dfrac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\,EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step #5 :-
Compute [D] Matrix

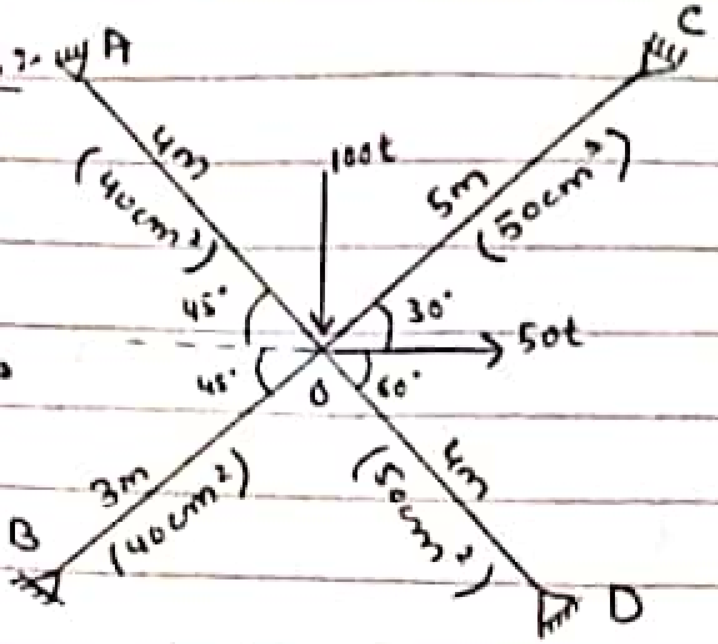
$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = [64.42]\ \frac{1}{EI}$

Problem # 03 :-



E = 2000 t/cm²

Sol: For A:

$$\sin 45° = \frac{P}{h} = \frac{P}{4}$$

$\Rightarrow$ $P = 2.828$ m

$$\cos 45° = \frac{b}{4}$$

$\Rightarrow$ $b = 2.828$ m

For B :-

$$\sin 45° = \frac{P}{3}$$

$\Rightarrow$ $P = 2.12$ m

$$\cos 45° = \frac{b}{h}$$

$\Rightarrow$ $b = 2.12$ m

For C :-

$$\sin 30° = \frac{P}{h = 5}$$

$\Rightarrow$ $P \Rightarrow 2.5$ m

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33 m$$

Now $EA_{(A)} = 2000 \times 40 = 80{,}000\, t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\, t$

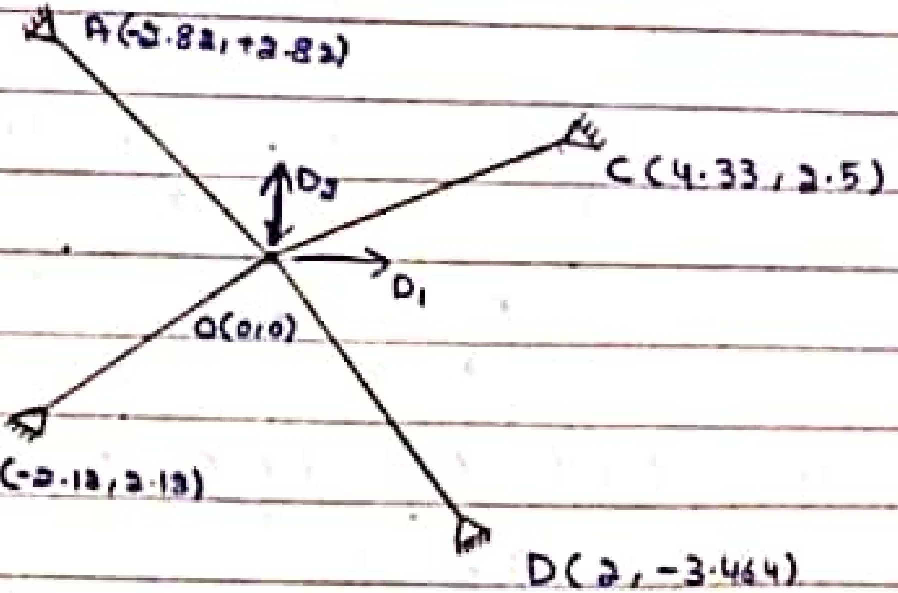
$EA_{(C)} = 2000 \times 50 = 100{,}000\, t$

$EA_{(D)} = 2000 \times 50 = 100{,}000\, t$

Step#01:- K.I

$$K.I = 2j - r$$
$$= 2(5) - 8 = 2^\circ$$

Step#02: Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step#03: $[AMD]_{4\times2}$ & $[S]_{2\times2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

Now $S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$

$$= \frac{80,000}{400^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2 + \frac{100,000}{(500)^3} \times (-433)^2$$

$$+ \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.167 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_j)$$

$$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212)$$

$$+ \frac{100,000}{(500)^3} \times (-433)(0-250) + \frac{100,000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1K'$

$$AMD = \frac{EA}{L^2} (Y_k - Y_j)$$

$$AMD_{12} = \frac{80,000}{400^2} (-282) = -141$$

$$AMD_{22} = \frac{80,000}{300^2} (212) = 188.44$$

$$AMD_{32} = \frac{100,000}{500^2} (-250) = -100$$

$$AMD_{42} = \frac{100,000}{400^2} (346) = 216.25$$

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3} (Y_k - Y_j)^2$

$$= \frac{80,000}{400^3} (-282)^2 + \frac{80,000}{300^3} (212)^2 + \frac{100,000}{500^3} (-250)^2$$

$$+ \frac{100,000}{400^3} (346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step #04: $[D] = [S]^{-1} \times [AD]$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step# 06:- $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.77 \end{bmatrix}$$

Scanned with CamScanner

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$