Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	SIR PIR MEHAR ALI	**Total Marks:**	20

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H\left(\frac{\pi}{4}\right)\right\|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ \ 1\ \ 2\ \ 3)$$	

Q No 1
(a)

Sol: The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} y(n-1) + 4k(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$ $k(1+4+4) \Rightarrow 2 \Rightarrow k$

$= k = 2/9$ total solution is

$$y(n) = \left[C_1 2^n + C_2 n^{2n} + \frac{2}{9}(-1)^n\right] u(n)$$

For initial condition we have $y(0)$

$y(0) = 1$, $y = 2$ then

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = 7/9$$

$$2C_1 + 2C_2 - 2/9 = 2$$

$\Rightarrow C_2 = 1/3$

Q No 1
part (b)

Sol:- The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y_h(n) = C_1 \frac{1^n}{2} + C_2 \frac{1^n}{5}$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) = 0.7y(0) = 0 \Rightarrow (1) = 1.4$$

Hence $C_1 + C_2 = 2$ and

$$\frac{1}{2} C_1 + \frac{1}{5} C_2 = 1.4 = 7/5$$

$$\Rightarrow C_1 + 2/5\, C_2 = 14/5$$

the evaluation yield

$C_1 = 10/3$, $C_2 = -4/3$

$$h(n) = 10/3 \left(1/2\right)^n - 4/3 \left(1/5\right)^n] u(n)$$

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The set response is

$$\theta(n) = \sum_{k=0}^{n} h(n-k)$$

$$\frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \left(\frac{1}{5} \left(5^{n+1} - 1\right)\right.\right.$$

$$u(n).$$

QNo2

(a):

Sol:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4(1+2z^{-1})} + \frac{3}{4}\frac{1}{1-2z^{-1}}$$

$$+ \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2}$$

By applying inverse transform.

$$X(n) = 1/8\,(-1)^{n}\,u(n) - 3/8\,u(n) + 1/2\,n\,u(n)$$

$$= \left[1/8\,(-1)^{n} + 3/8 + n/2\right] u(n)$$

QNo2

(b)

Sol:-

First we eleminate the negative power by Multiplying both numerator and denuntor by z^2 thus

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

The poles are X(z) are $p_1 = 1$ and $p_2 = 0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

To determine A_1 and A_2 is to multiply the equation by denumetor term $(z-1)(z-0.5)$

$$z = (z-0.5)A_1 + (z-1)A_2$$

$z = p_1 = 1$

$$1 = (1-0.5)A_1$$

thus we obtain the result $A_1 = 2$ and set $z = p_2 = 0.5$ so elemating A_1 we have

$$0.5 = (0.5-1)A_2$$

$$A_2 = -1$$

So the result of partial fraction is

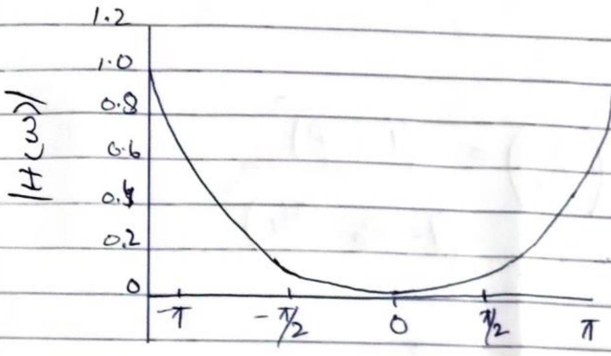
$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

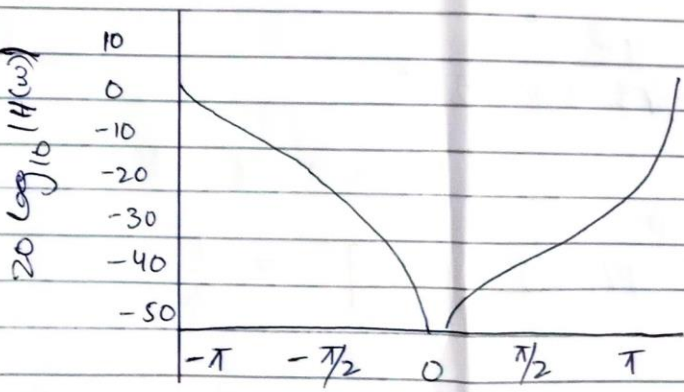
QNo3 (a):-

Sol:- At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$





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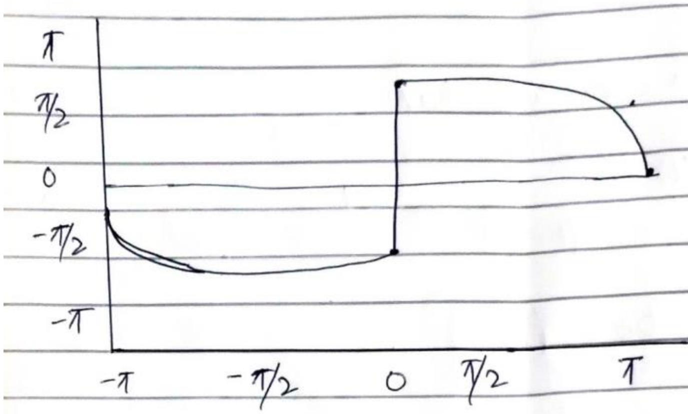

At $\omega = \pi/4$

$$H\left(\pi/4\right) = \frac{(1-p)^2}{\left(1-pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-p)^2}{1-p(\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = \frac{1}{2}$$

Q No 3

Part (b):-

Sol: By the filter requirments:-

Poles $P_{1,2} = re^{\pm j\pi/2}$ pass band center

Zeroes $Z_{1,2} = \pm 1$ Stopband Canter.

$$\Rightarrow H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)} = G \frac{z^2-1}{z^2+r^2}$$

By the filter requirement

$$H(\pi/2) = G \frac{-2}{-1+r^2} = 1$$

$$= G \frac{1-r^2}{2}$$

To set r use $H(4\pi/9) = 1/\sqrt{2}$

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ therefore

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

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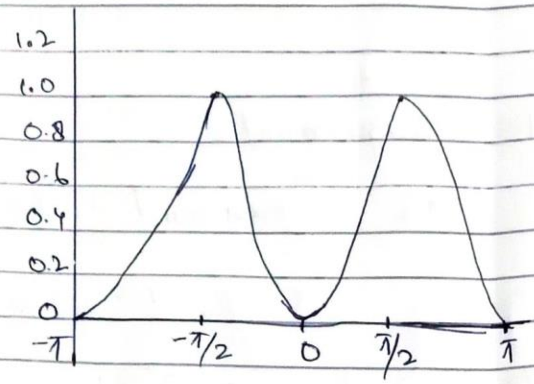
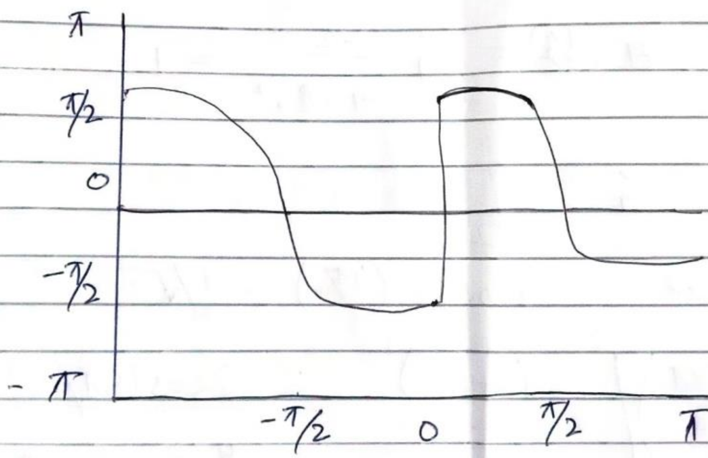




Q No 4

(a)

Sol:- The fourier transform of this sequence is

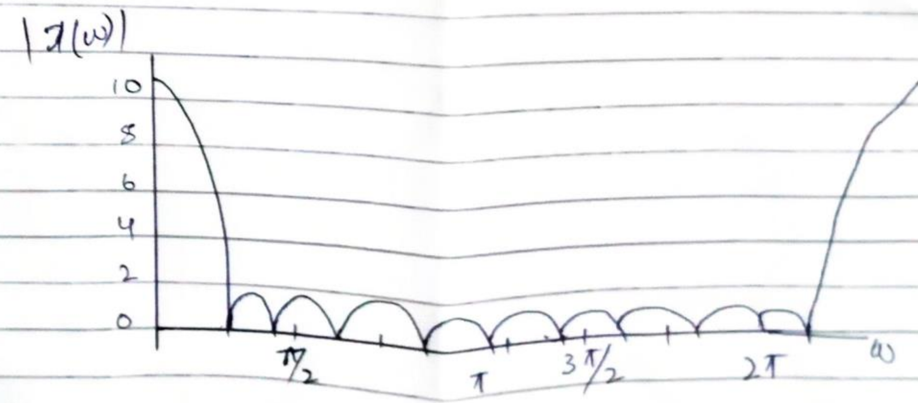
$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

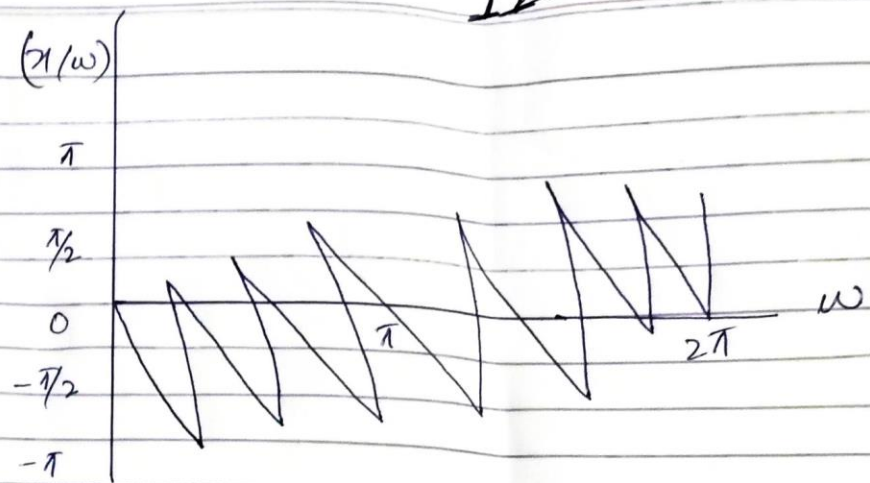
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega L}} =$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

$$X(K) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad K = 0, 1 - - - N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





If N is selected such that $N = L$ then the DFT becomes

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2 \text{ ---- } L-1 \end{cases}$$

thus there is only one nonzero value in DFT.

Q No 4

b) Sol:- $\omega_N^{k+N/2} = \omega_N^{k}$

Then Matrix can be extend by

$$\omega_N = \begin{bmatrix} \omega_N^0 & \omega_N^0 & \omega_N^2 & \omega_N^{N-1} \\ \omega_N^0 & \omega_N^2 & \omega_N^4 & \omega_N^{2(N-1)} \\ \omega_N^0 & \omega_N^2 & \omega_N^4 & \omega_N^{N-1} \\ \omega_N^0 & \omega_N^{2N-1} & \omega_N^{N-1(N-1)} & \omega_N^{N-1\,N-1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega_N^1 & \omega_N^2 & \omega_N^3 \\ 1 & \omega_N^2 & \omega_N^0 & \omega_N^4 \\ 1 & \omega_N^1 & \omega_N^3 & \omega_N^4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then $X_4 = \omega_N \times 4 = \begin{bmatrix} 4 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$ Ans