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Question 1

a) Determine the response $y(n)$, $n \geq 0$ of the system described by the second order difference equation:

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$. And the initial conditions are $y(-1) = y(-2) = 0$.

Solution:

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence

$$y_h(n) = C_1 2^n + C_2 2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

Subsituting this solution into the difference equation we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) =$$

$$(-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $k(1+4+4) = 2$

$k = 2/9$ The total solution is

$$y(n) = [C_1 2^n + C_2 2^n + 2/9(-1)^n] u(n)$$

From the initial conditions

We obtain, $y(0) = 1$, $y(1) = 2$

Than

$$c_1 + 2/9 = 1$$

$\Rightarrow c_1 = 7/9$

$\Rightarrow 2c_1 + 2c_2 - 2/9 = 2$

$\Rightarrow c_2 = 1/3$ Ans.

b) Determine the impulse response and unit step response of the system described by the difference equation.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Solution:

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = 1/2, 1/5$ Hence

$$y_h(n) = c_1 \, 1/2^n + c_2 \, 1^n/5$$

with $x(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0$$

$\Rightarrow y(1) = 1.4$

Hence, $c_1 + c_2 = 2$

And,

$$1/2 \, c_1 + 1/5 = 1.4$$

$1.4 = 7/5$

$c_1 + 2/5\, c_2 = 14/5$

These equations yield

$c_1 = 10/3$, $c_2 = -4/3$

$$h(n) = \left[\frac{10}{3}(1/2)^n - \frac{4}{3}(1/5)^n\right] u(n)$$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \frac{10}{3}\sum_{k=0}^{n}(1/2)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}(1/5)^{n-k}$$

$$s(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5k$$

$$s(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1} - 1)\, u(n) - \frac{1}{3}\left(\frac{1}{5}\right)^n$$

$$(5^{n+1} - 1)\, u(n) \quad \text{Ans.}$$

QUESTION 2

a) Determine the causal signal $x(n)$ having the Z-transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution:

Taking inverse and z-transform

$$\frac{A}{[1-2z^{-1}]} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence

$x(n) = [4(2)^n - 3 - n]\, u(n)$ Ans

b) Perform the circular convolution of the following two sequences. Solve the problem step by step.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Solution:

Each sequence consist of four non zero points for the purpose of illustrating the operations involved in circular convolution it is desired to graph each Sequence as points on a circle. Thus the sequences $x_1(n)$ and $x_2(n)$ are graphed as illustrated we note that the sequences are graphed in a counterclockwise direction on a circle.

Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with

⇒ $x_1(n)$ with $x_2(n)$ as specified. Begning with $m=0$. we have

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]_N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle. The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point. Finally we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

For $m=1$, we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

it is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter clockwise by one unit in the time.

⇒ This rotated sequence multiplies $x_1(n)$ to yield the product sequence also finally we sum the values in the product sequence to obtain $x_3(1)$ Thus.

$$x_3(1) = 16$$

For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.

$x_1(1) = 1$

$x_1(2) = 2$ $x_1(n)$ $x_1(0) = 2$

$x_1(3) = 1$

$x_2(1) = 2$

$x_2(2) = 3$ $x_2(n)$ $x_2(0) = 1$

$x_2(3) = 4$

(A)

$x_2(3)$

$x_2(2) = 3$ $x_2(-n)_4$ $x_2(0) = 1$

$x_2(1) = 2$

Folded sequences

4

6 $x_1(n)(-n)_4$ 2

2

Product sequence

(B)

$x_2(0) = 1$

$x_2(3) = 4$ $x_2[(1)-(n)]_4$ $x_2(1) = 2$

$x_2(2) = 3$

Folded sequence rotated by ~~two~~ one unit in time

1

8 $x_1(n)\, x_2(1-n)_4$ 4

3

Product sequence

(C)

Scanned with CamScanner

$x_2(1)=2$

$x_2(0)=1$ $\quad x_2(2-1)\ 4 \quad$ $x_2(2)=3$

$x_2(3)=4$

2

2 $\quad -x_1(n)\ x_2(2-n)\ 4 \quad$ 6

4

Folded sequence rotated by two unit in time

Product sequence

(D)

$x(2)=4$

$x_2(1)=2$ $\quad x_2(3)\ 4 \quad$ $x_2(3)=4$

$x_2(0)=1$

3

4 $\quad x_1(n)x_2(3-n) \quad$ 8

1

Folded sequence rotated by three unit in time

Product sequence

QUESTION 3

a) A two - pole low pass filter has the system response.

$$H(z)=\frac{b_0}{(1-Pz^{-1})^2}$$

Determine the value of b_0 and P such —

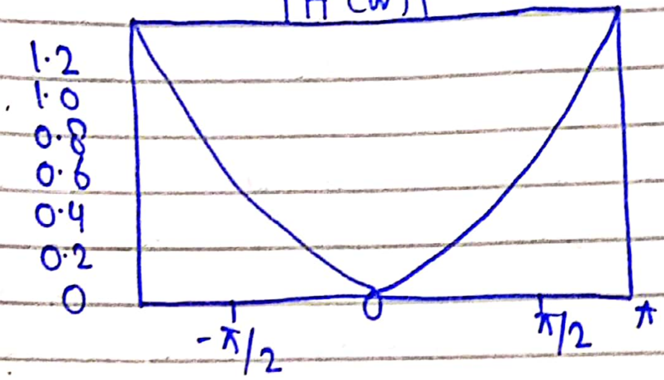
Solution:

At $\omega = 0$ we have

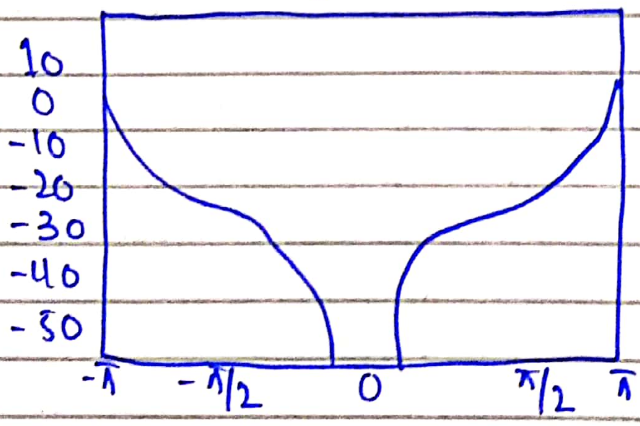
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence $b_0 = (1-p)^2$

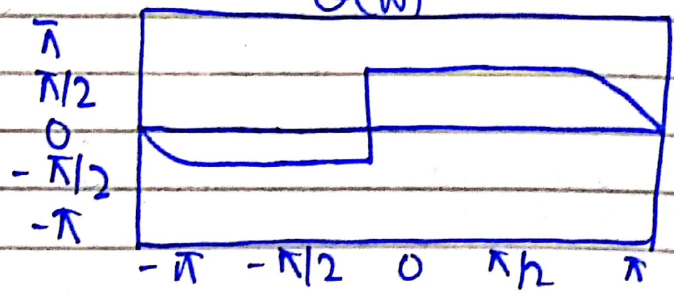
$|H(\omega)|$



$20 \log_{10} |H(\omega)|$



$\Theta(\omega)$



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At $w = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-p\,e^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{[1-p\cos(\pi/4)+jp\sin(\sqrt{2})^2]}$$

$$= \frac{(1-p)^2}{[1-p(\sqrt{2}+jp\sqrt{2})^2]}$$

Hence

$$= \frac{(1-p)^4}{[(1-p(\sqrt{2})^2+p^2/2]^2}$$

$$= 1/2$$

$\Rightarrow$ Equivalently

$$\sqrt{2}\,(1-p)^2 = 1+p^2-\sqrt{2}p$$

The system function for desired filter

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2} \quad \text{Ans}$$

b) Design a two-pole bandpass filter that has the center of its passband at $w = \pi/2$, zero in its frequency response characteristics at $w=0$ and $w=\pi$ and its magnitude response is $\frac{1}{\sqrt{2}}$ at $w = 4\pi/9$

Solution:-

The filter must have poles at

$P_{1,2} = re$

And zero at $z = 1$ and $z = -1$
Consequently the same system function is

$$\Rightarrow H(z) = G\,(z-1)(z+1) / (z-jr)(z+jr)$$

$$\Rightarrow H(z) = \frac{G\,z^2 - 1}{z^2 + r^2}$$

The gain factor is determined evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

$$H = (\pi/2) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$ we have

$$\left|H(4\pi/9)\right|^2 = \frac{(1-r^2)^2}{4}\;\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

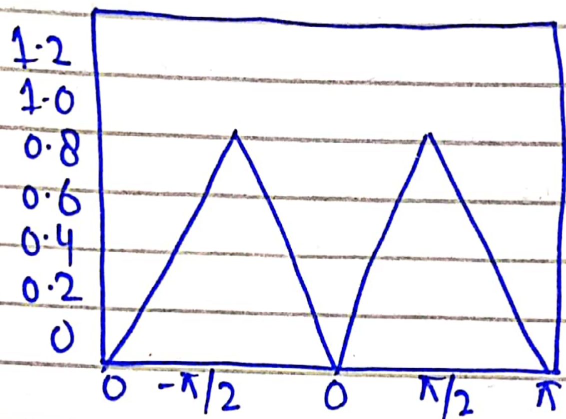
$\Rightarrow$ Equivalently

1.94 $(1-r^2)^2 = 1 - 1.88r^2 + r^4$ the value of $r^2 = 0.7$ satisfies this equation therefore the system function for the desired filter is

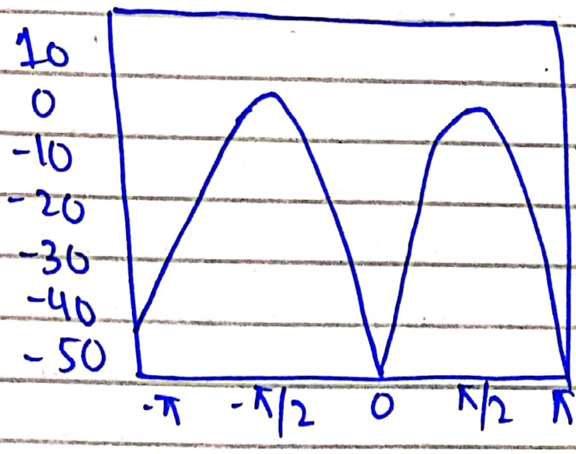
$$\Rightarrow H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

its frequency response is illustrated

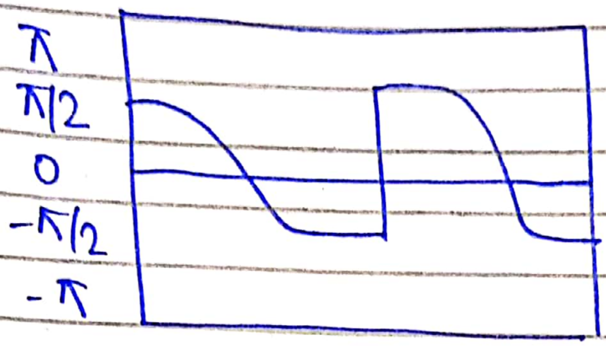
$|H(w)|$



$20 \log(H(w))$



Phase radiation



Magnitude and Phase response of a simple bandpass filter is $H(z) = 0.25\left[(1-z^{-2})/(1+0.7z^{-2})\right]$

Ans.