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Assignment :- (1)

Subject :- Advance Mechanics
of Material.

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①

Assignment #①

Q1) Explain the following topic in detail with neat sketches in detail.

① Application of Mohr's Circle to the three-~~dimension~~ σ .

② Dimensional Analysis of Stress.

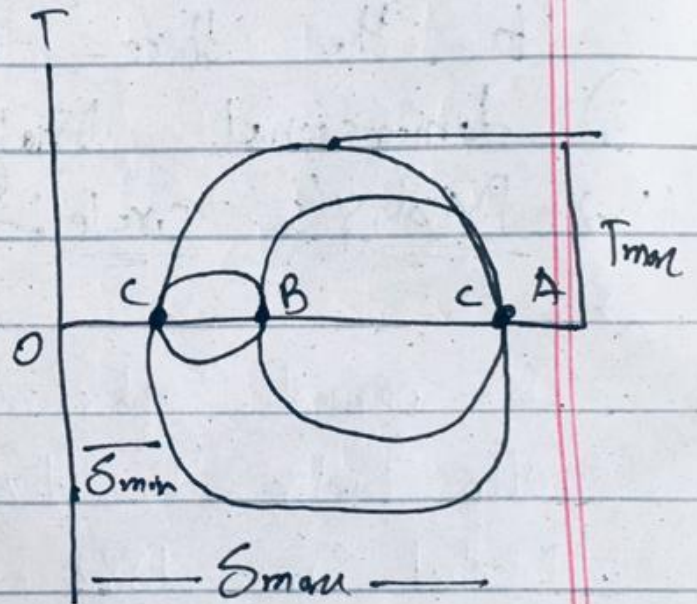
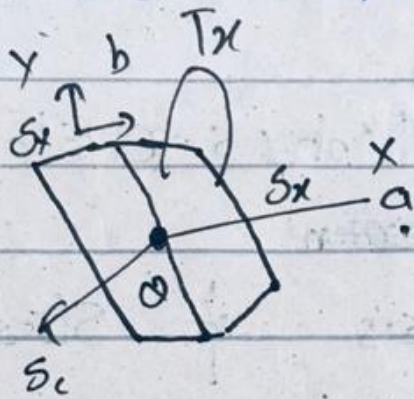
(Ans) Mohr's Circle:-

→ Mohr's circle is named after a German civil Engineer Otto Mohr - He developed the graphical Method in 1882.

→ Mohr circle is a graphical representation of a given general state of a point - it is graphical method used for evaluation of principal stresses maximum shear stress normal and tangential stresses on any given plane.

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→ The Mohr's circle describe the normal and shear stress acting on plane of all possible orientation through a point in the rock.

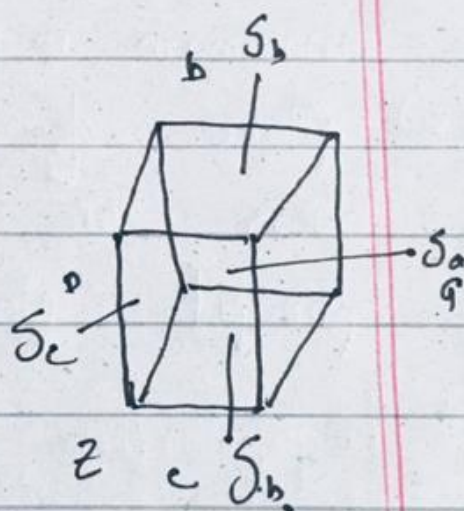
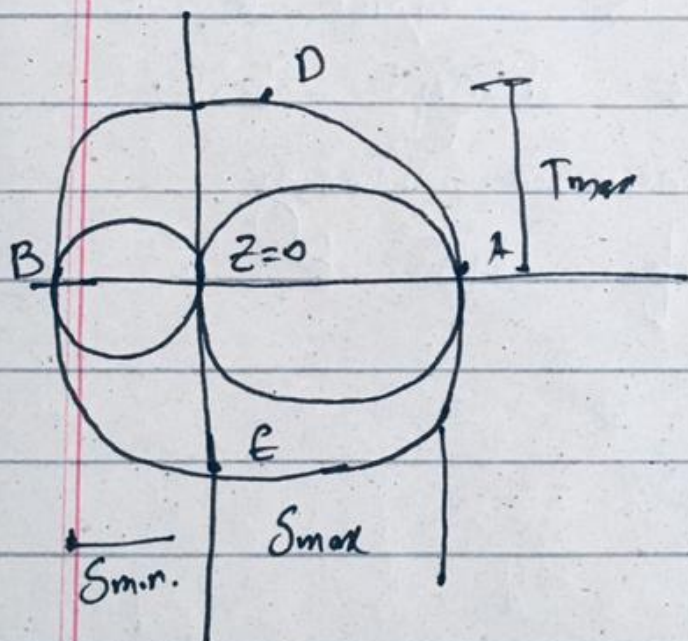


* transformation of stress for an element rotated around a principal axis may be represent by Mohr's circle -
→ point A, B and C represent the principal stresses on the principal planes (shearing stress is zero)

3.

- The three dimensional represent the normal and shearing stresses for rotation around each principal axis.
- Radius of the largest circle yield the maximum shearing stress

$$\tau_{max} = \frac{1}{2} |\sigma_{max} - \sigma_{min}|$$



- in the case of plane stress, the axis perpendicular to the plane of stress is a principal axis (shearing stress equal zero)

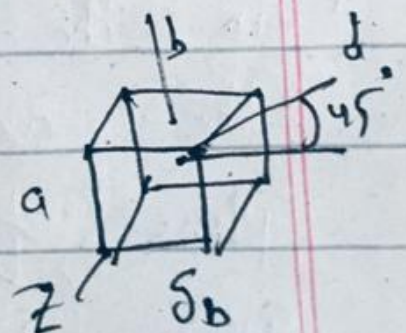
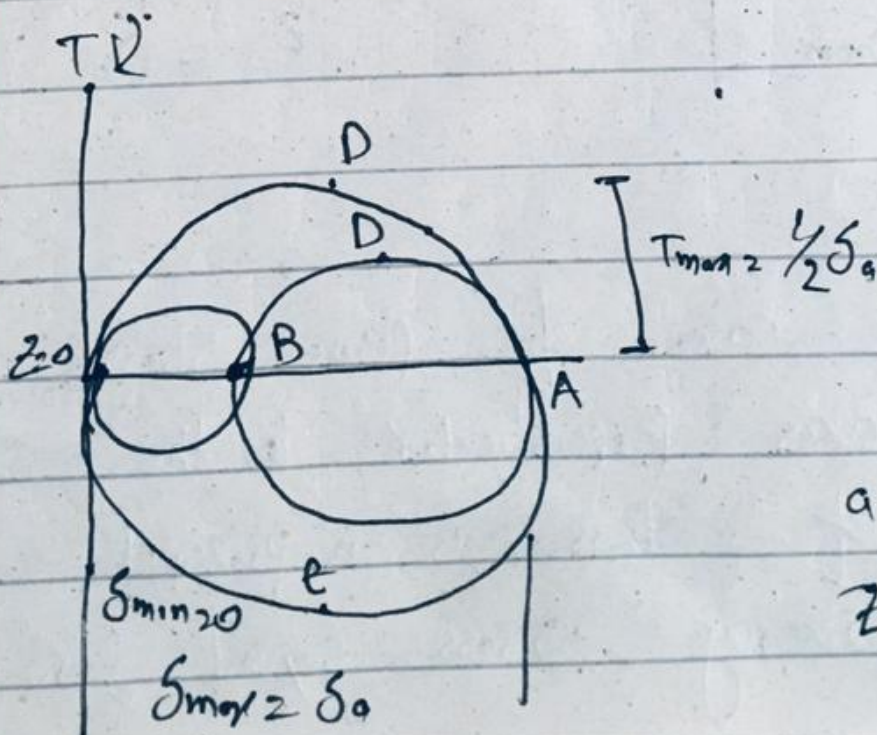
(4)

→ If the point A and B (representing the principal planes) are on opposite sides of the origin than

(a) the corresponding principal stresses are the maximum and minimum normal stresses for the element

(b) the maximum shearing stress for the element is equal to the maximum in plane shearing stress

(c) plane of maximum shearing stress are at 45° to the principal plane



(5)

- if A and B are on the same side of the origin (i.e. have the same sign) then
- (a) the circle defining σ_{max} , σ_{min} and τ_{max} for the element is not in the circle corresponding to the transformation within the plane of stress -
- (b) maximum shearing stress for the element is equal to half of the maximum stress -
- (c) plane of maximum shearing stress are at 45 degree to the plane -

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(3) Simple Bending and pure Bending'

(Ans) Bending Stresses:-

The stresses introduced by bending moment are known as Bending Stresses-

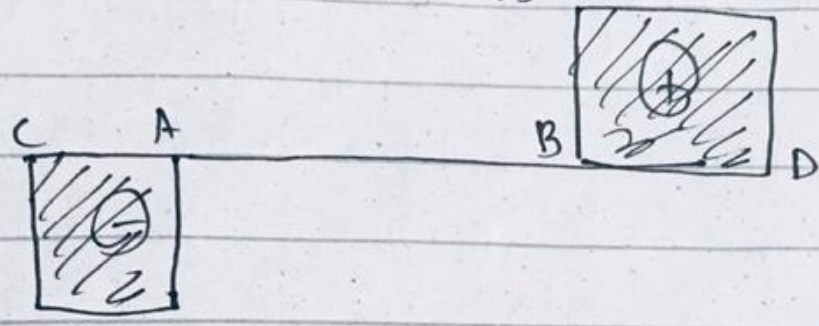
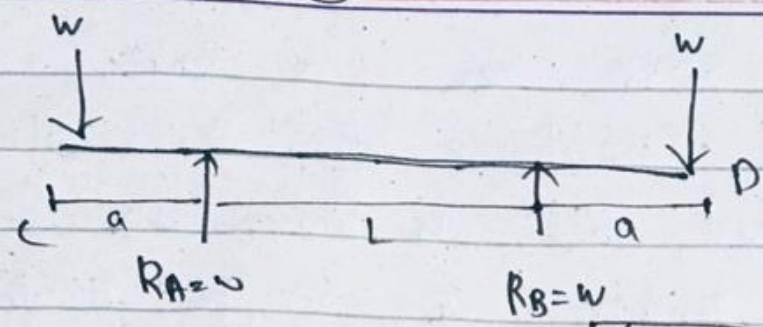
→ Pure Bending and Simple Bending

If a length of beam is subjected to constant bending moment and no shear force (zero shear force) stresses will be setup in that length

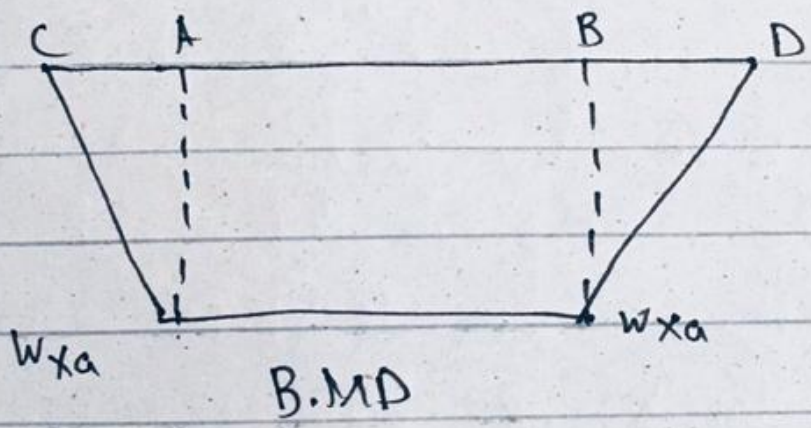
of beam due to B.M only that length of beam is

said to be in pure Bending or Simple Bending and the stresses setup in that length are known as bending stresses.

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S: F: D.



- There is no shear force b/w A and B as shown.
- But Bending Moment is constant b/w A and B.
- Beam is subjected to constant B.M b/w A and B and

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no shear force is there
this condition of beam b/w
A and B is known as
pure bending or simple bending

(9)

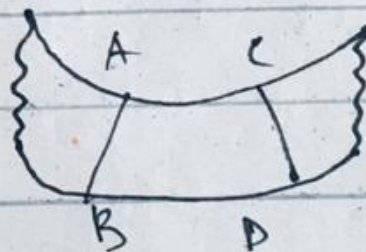
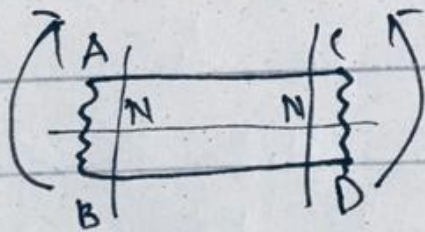
(1) Assumption made in theory of Pure Bending,

(1) The material of the Beam is homogenous and isotropic.

Homogenous \rightarrow Material is of some kind throughout.

(2) In the value of young's Modulus of elasticity is same in tension as well as in compression.

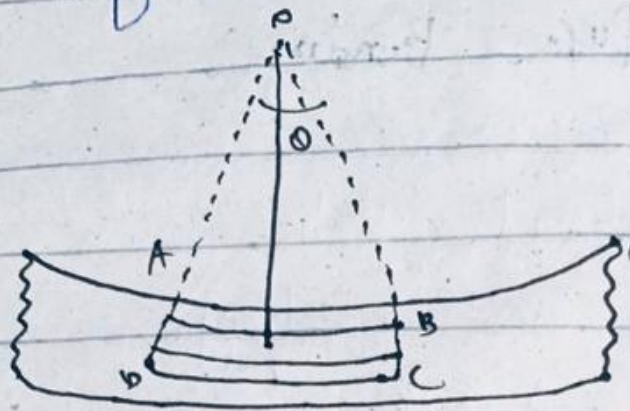
(3) The transverse section of beam which were plane before bending remain plane after bending also.



(4) The Beam is initially straight and all longitudinal fibers bend into

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crossed area with common
center of curvature



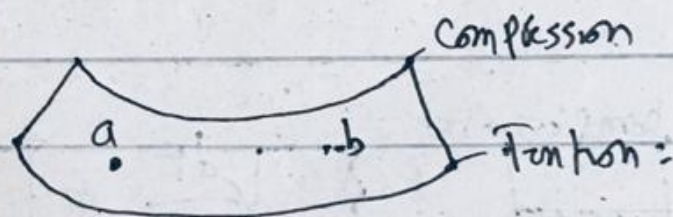
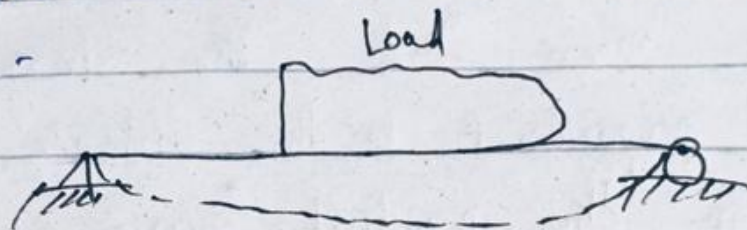
- (5) The radius of curvature is large as compared with dimension of cross section.
- (6) Each layer of the beam is free to expand or contract independently of the layer above or below it.

(5) Classic flexure Equation 2

(Axis) Stresses caused by the Bending moment etc known as flexural or

Bending Stresses -

consider a beam to be loaded as shown -



consider a fiber at a distance y from neutral axis because of the beam

curvature as the effect of

Bending moment, the fiber is stretched by an amount of cd

Since the curvature of the

beam is very small, abd

and oba are considered as

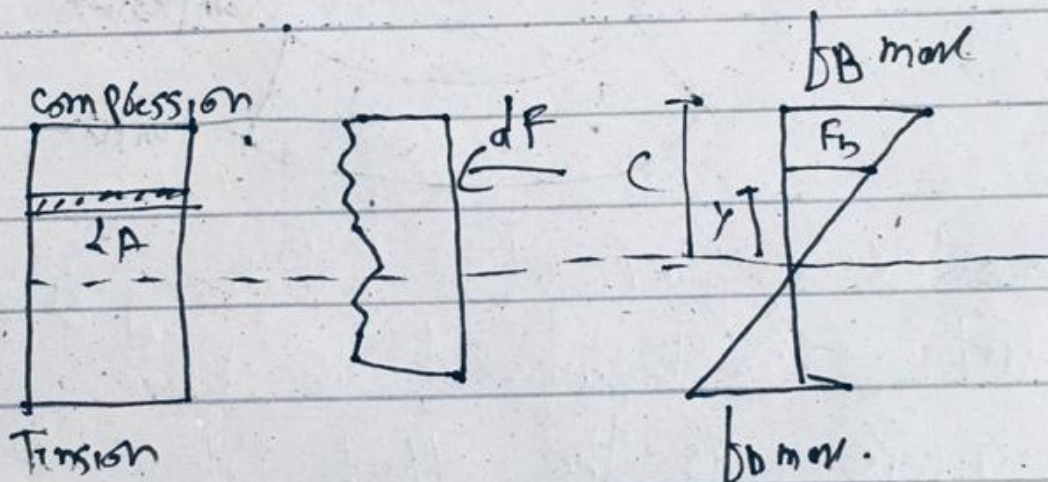
similar triangle the strain on this fiber is.

$$\epsilon = \frac{cd}{ab} = \frac{y}{r}$$

by Hook Law $\epsilon = \frac{\sigma}{E}$ then

$$\frac{\sigma}{E} = \frac{y}{r} \therefore \sigma = \frac{y}{r} E$$

which mean that the stress is proportional to the distance y from the neutral axis.



consider a differential area dA at a distance y from N.A. the force acting over the Area is

$$dF = \sigma dA = \frac{y}{r} E dA = \frac{E}{r} y dA$$

The resultant of all elemental moment about N.A. must be

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equal to bending moment on the section -

$$M = \int dm = \int y dF = \int y \left(\frac{E}{\rho} y dA \right)$$

$$M = \frac{E}{\rho} \int y^2 dA$$

but

$$\int y^2 dA = I \text{ then:}$$

$$M = \frac{E I}{\rho} \text{ or } \rho = \frac{E I}{M}$$

Substituting $\rho = E y / f_b$

$$\frac{E y}{f_b} = \frac{E I}{M}$$

then

$$f_b = \frac{M y}{I}$$

and

$$f_b = \frac{M c}{I}$$

The Bending Stress due to Beam curvature is

$$f_b = \frac{M c}{I} = \frac{E I}{\rho I} c / \rho$$

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$$\delta = \frac{Ec}{\rho}$$

The Beam curvature is

$$k = \frac{1}{\rho}$$

where ρ is the radius of curvature of the beam in (mm) (in) M is the Bending moment σ is the flexural stress in MPa I is the Centroidal moment of inertia and c is the distance from the neutral axis to the outermost fiber is mm -

⑥ Section Modulus 2

Section modulus

is a geometric property for a given cross-section used in the design of beam or flexural member. other geometric property used in the design include area for tension and shear, radius of gyration for compression and moment of inertia and polar moment of inertia for stiffness.

Section Modulus formula 2

$$b_b(\text{mod}) = \frac{Mc}{I} = \frac{M}{I/c}$$

The ratio I/c is called section modulus and is usually denoted by S with units of mm^3 . The maximum bending stress may be then written as -

$$b_b(\text{max}) = \frac{M}{S}$$

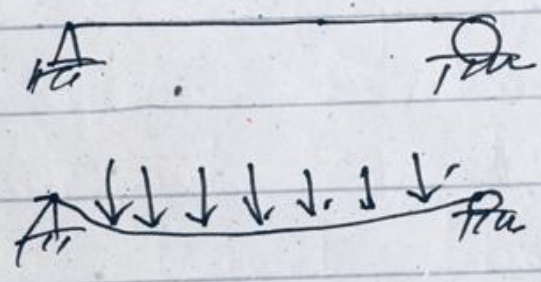
$$S_{min} = \frac{M_{max}}{\sigma_{all}}$$

Some of the table are -

Shape	S. mm ³
W410 x 38.8	637
W360 x 32.9	474
W310 x 38.7	549
W250 x 44.8	535
W200 x 46.1	448

⑦ Application of Bending Equation in any object.

A bending moment is a measure of the Bending effect due to forces acting on a beam - it is measured in term of force and distance -



- The Bending moment equation include the Bending moment resisted by the beam section along the length of the beam -
- Using these equation the type and size of a member of a given material can be easily determined -
- Another application of shear and moment equation is that the

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Deflection can be easily determined using either the moment area method or the conjugate beam method -

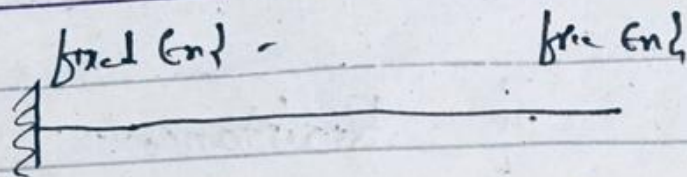
⑤ Moment of Resistance²

When a beam bends under load the horizontal fiber will change in length. In technical term it is referred to as the internal moment of resistance - the tensile and compressive stresses result in a turning effect about the neutral axis these are called moment M_T and M_C respectively -

In technical term, the maximum bending moment a section can resist is called moment of resistance -

Whenever a force acts at a distance from fix point a moment is produced and this moment tries to bend it from that point this is moment -

(20)

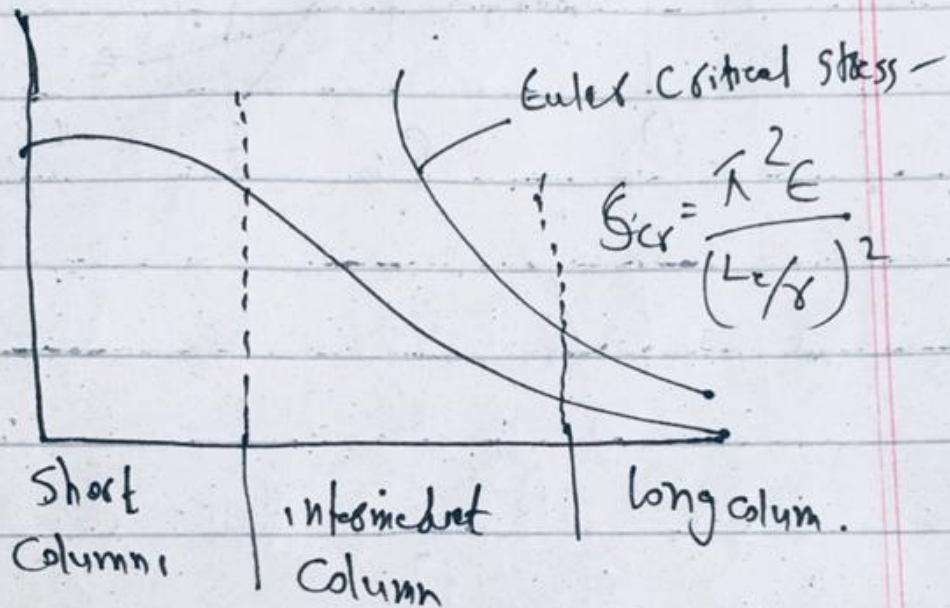


Now consider a Beam, when a force is Applied at the free end - moment is produce. As the force is increased - moment increase -

At a certain value of moment the beam fails at the fixed End which means that the moment at the section (fixed End) has reached its max value -

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(a) Design of Column Under Centric Load.



Experimental data demonstrate -

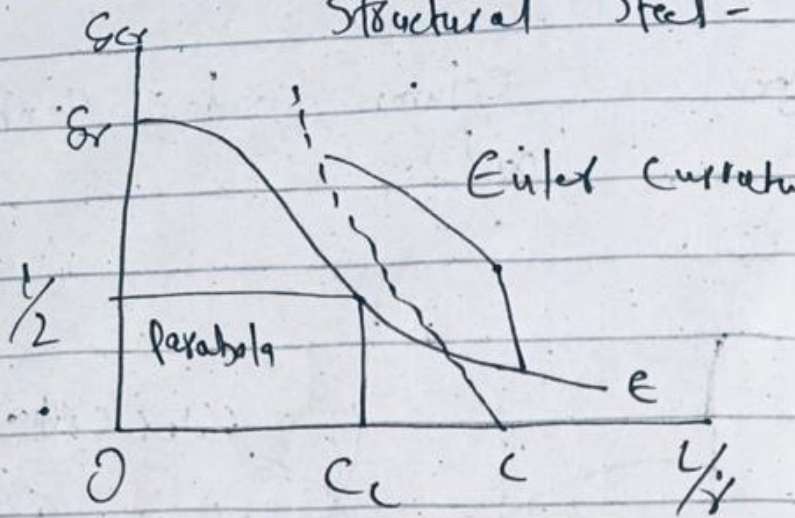
→ for large L_e/r σ_{cr} follow Euler formula and depends upon E but not σ_y -

→ for small L_e/r σ_{cr} is determined by the yield strength σ_y and not E .

→ for intermediate L_e/r σ_{cr} depend on both σ_y and E .

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Structural Steel -



for $L_e/r > c_c$

$$\sigma_{cr} = \frac{\pi^2 E_c}{(L_e/r)^2} \quad \sigma_{all} = \frac{\sigma_{cr}}{F_s}$$

$$F_s = 1.92$$

for $L_e/r > c_c$

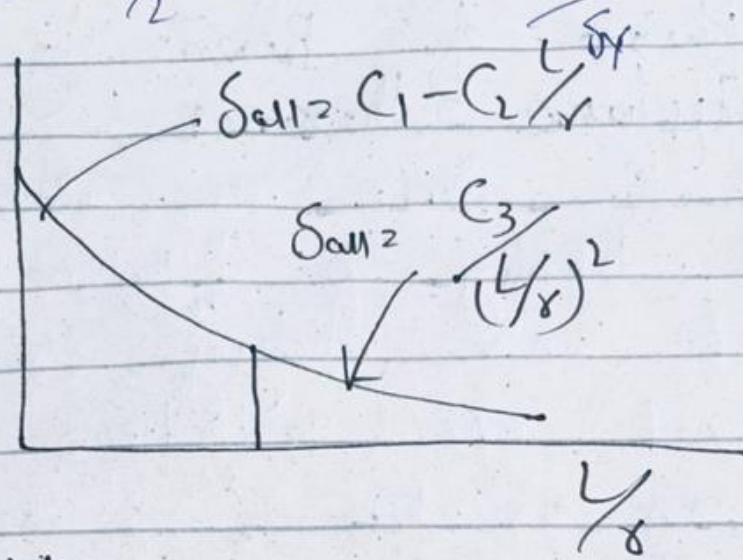
$$\sigma_{cr} = \sigma_y \left[1 - \frac{(L_e/r)^3}{2c_c^3} \right] \quad \sigma_{all} = \frac{\sigma_{cr}}{F_s}$$

$$F_s = \frac{5}{3} + \frac{3}{8} \frac{L_e/r}{c_c} - \frac{1}{8} \left(\frac{L_e/r}{c_c} \right)^3$$

at $L_e/r = c_c$

(23)

$$\sigma_{cr} = \frac{1}{2} \sigma_y \cdot \left(C_1^2 + 2K^2 \frac{L}{\delta} \right)$$



Aluminium

$$L/d \leq 55$$

$$\sigma_{all} = [30.7 - 0.22 (L/d)] \text{ ksi}$$

$$[212 - 1.589 (L/d)] \text{ MPa}$$

$$L/d > 66$$

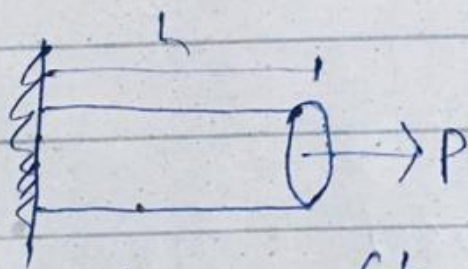
$$\sigma_{all} = \frac{54000 \text{ ksi}}{(L/d)^2} = \frac{372 \times 10^3 \text{ MPa}}{(L/d)^2}$$

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10) Deflection of Beams by Castigliano's Theorem²

Ans. The theorem also allow for the determining of deflection for object with changing cross sectional area. Castigliano's theorem is given as where δ is the deflection U is the strain energy and P is the force or torque at a certain point.

→ Different loading condition required strain energies for each loading



$$U = \int_0^L \frac{P^2 dx}{2EA}$$

where P is the load E is the material Young modulus A is the cross sectional area, and L is the length.