DEPTT : "CIVIL ENGINEERING"

EXAM:

MID TERM

NAME:

MUHAMMAD ILYAS

ID:

"7956"

SECTION:

"B"

PAPER:

MOS 2

SEMESTER:

"4th"

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24

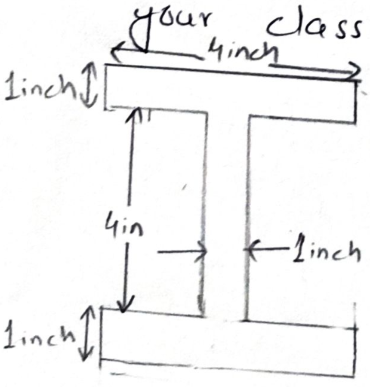
SUBMITTED TO:

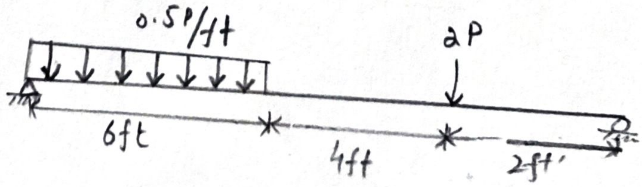
ENGR. SIR MUHAMMAD SAQIB

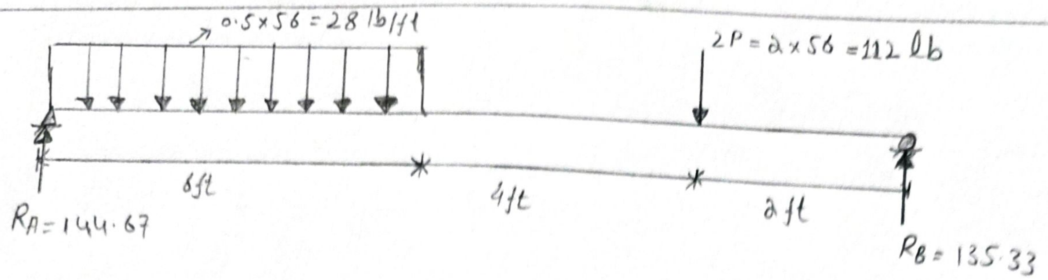
1)⇒ Construct the Mohr's Circle diagram and find the principal stress and maximum in plane shear stress for the stress area of a Point "C" Located at the center of uniformly distributed load and One(1) Inches below the top fiber of a beam cross Section as Shown in figure. However to Construct the Mohr's Circle it is necessary to draw the shear stress and flexural stress variation diagram for maximum shear force and bending moment respectively. Compare the resultant obtained from the Mohr's circle with the stress transformation equation.

Hint: To Calculate the shear stress in beam cross Section the moment of Inertia must be known.

⇒ Where P is the last two digits of your class registration number is Pounds ⇒ 56lb







$\curvearrowright \sum M_B = 0$

$R_A \times 12 - (28 \times 6) \times (6+3) + 112 \times 2 = 0$

$R_A = 144.67$ lb

$\curvearrowright \sum M_A = 0$

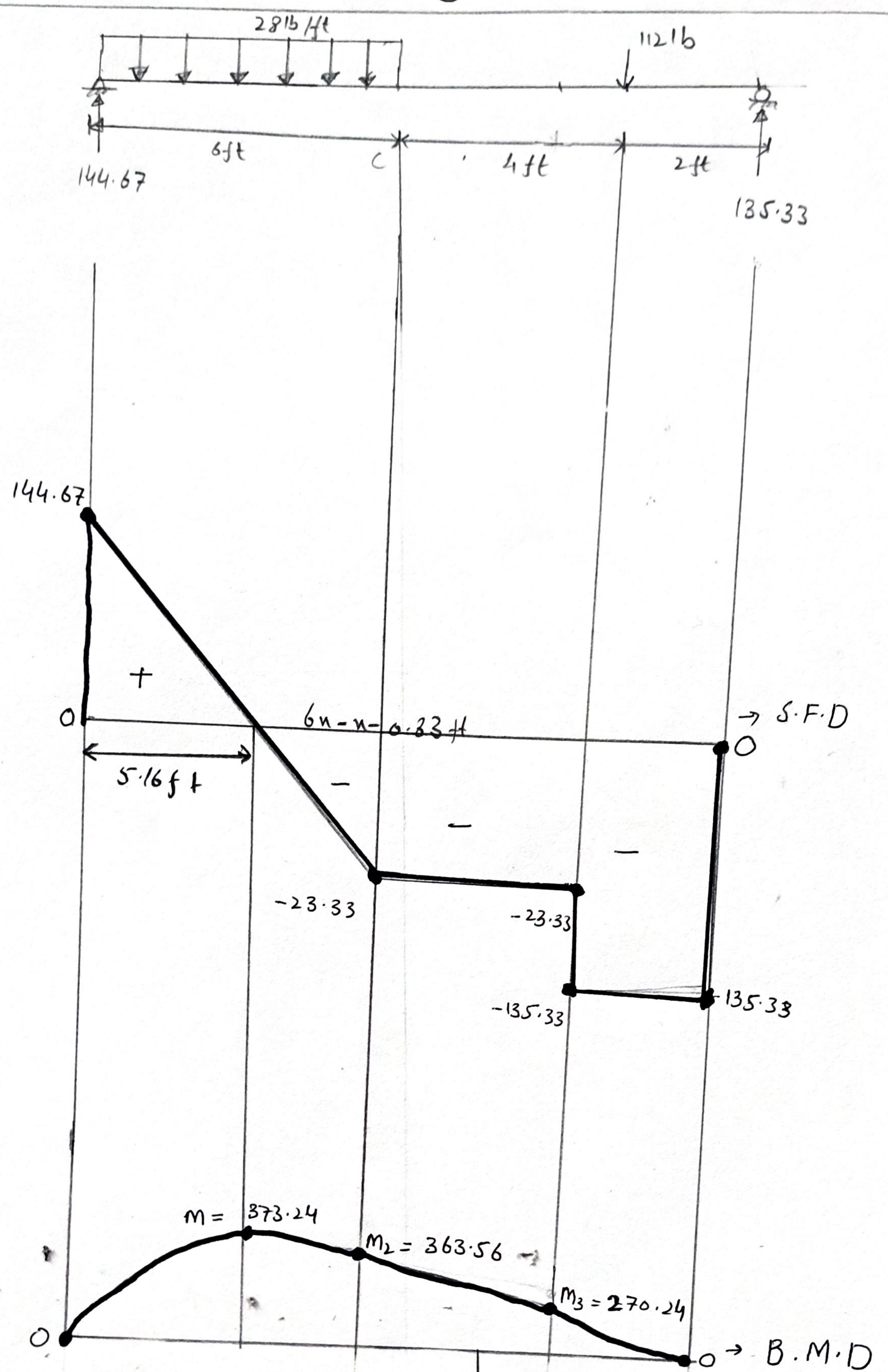
$-R_B \times 12 + 112 \times 10 + (28 \times 6) \times 3 = 0$

$R_B = 135.\overset{33}{\cancel{34}}$ lb

$\sum F_y = 0$

$28 \times 6 + 112 = 144.67 + 135.34$

$280 = 280$ (Checked)



$$\Rightarrow \frac{x}{144} = \frac{6-x}{23.33}$$

$$\Rightarrow 23.33x = 864 - 144x$$

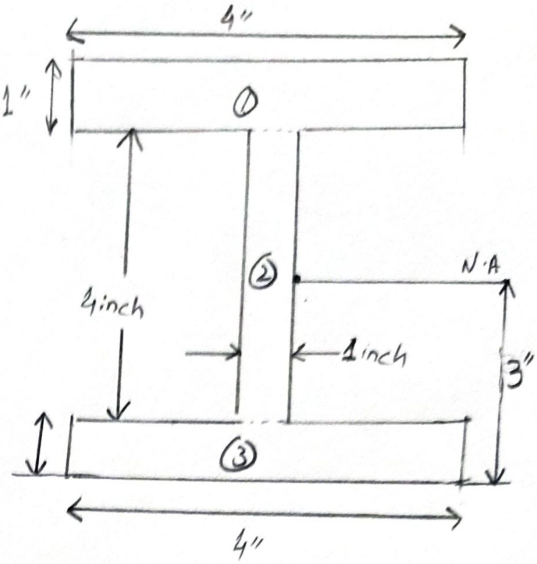
$$\Rightarrow 167.33x = 864$$

$$x = 5.16 \text{ ft}$$

$$+\circlearrowleft M_{5.16} = 0$$

$$M_{5.16} - 144.67 \times 5.16 + (28 \times 5.16) \times 5.16/2 = 0$$

$$M_{5.16} = 373.24 \text{ lb-ft}$$



$$\bar{y} = \frac{y_1 A_1 + y_2 A_2 + y_3 + A_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{5.5 \times (4\times1) + 3\times(4\times1) + 0.5\times(4\times1)}{4+4+4}$$

$$\bar{y} = 3 \text{ in}$$

$$\left(I_x = I_{x_1} + I_{x_2} + I_{x_3} \right) \rightarrow eq^{(1)}$$

For I_{x_1}

$A_1 = 4\times1$

$A_1 = 4 in^2$

$I_1 = \frac{bh^3}{12}$

$I_1 = \frac{4\times1^3}{12}$

$I_1 = 1/3 \; in^4$

$d_y = y - \bar{y}$

$d_{y_1} = 5.5 - 3$

$d_{y_1} = 2.5 in$

$I_{x_1} = I_1 + A_1 d_{y_1}^2$

$I_{x_1} = 1/3 + 4\times2.5^2$

$\boxed{I_{x_1} = 25.33 \; in^4}$

For I_{x_2}

$A_2 = 4\times1$

$A_2 = 4 \; in^2$

$I_2 = \frac{b\times h^3}{12}$

$I_2 = \frac{1\times4^3}{12}$

$I_2 = 16/3 \; in^4$

$d_2 = y - \bar{y}$

$= 3 - 3$

$= 0$

$I_{x_2} = I_2 + A_2 d_{y_2}^2$

$I_{x_2} = 16/3 + 0$

$\boxed{I_{x_2} = 5.33 \; in^4}$

For I_{x_3}

$A_3 = 4\times1$

$A_3 = 4 \; in^2$

$I_3 = \frac{4\times1^3}{12}$

$I_3 = 1/3 \; in^4$

$d_{y_3} = \bar{y} - y$

$d_3 = 3 - 0.5$

$d_3 = 2.5 in$

$I_{x_3} = I_3 + A_3 d_{y_3}^2$

$I_{x_3} = 1/3 + 4\times2.5^2$

$\boxed{I_{x_3} = 25.33 \; in^4}$

eq (i)

$I_x = 25.33 + 5.33 + 25.33$

$\boxed{I_x = 55.99} \rightarrow \underline{\mathbf{56\ in^4}}$

"Shear Stresses Analysis"

$V_{max} = 144 lb$

$I = 56\ in^4$

$\tau = \frac{VQ}{Ib}$, $Q = \bar{y}A$

Case: 01: τ at the top fiber

$Q = \bar{y} \times A$

$Q = y \times 0$

$Q = 0$

$\tau = \frac{144 \times 0}{56} = 0$

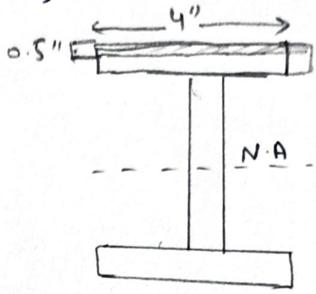
Case: 2 τ at the (0.5) in below the top fiber.

$$\tau = \frac{V \times Q}{I \times b}$$

$Q = \bar{y} \times A$

$= (2 - 0.75) \times (0.5 \times 4)$

$Q = 5.5 \, in^3$



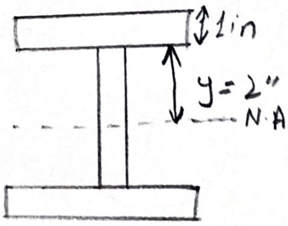
$$\tau = \frac{144 \times 5.5}{56 \times 4}$$

$\tau = 3.54 \, lb/in^2$

Case #3 τ at 1 in below the top fiber.

$Q = \bar{y} A$

$Q = 10 \, in^3$



$\Rightarrow b = 4 \, in$

$$\tau = \frac{VQ}{Ib} = \frac{144 \times 10}{56 \times 4}$$

$\tau = 6.43$ psi

Now

$b = 1"$

$Q = 10 \, in^3$

$$\tau = \frac{144 \times 10}{56 \times 1} = 25.71 \text{ Psi}$$

Case : 4 τ at centriodal axis

$b = 1''$

$Q = Q_1 + Q_2$

$Q = \bar{y}_1 A_1 + \bar{y}_2 A_2$

$Q = 2.5 \times (4 \times 1) + 1 \times (2 \times 1)$

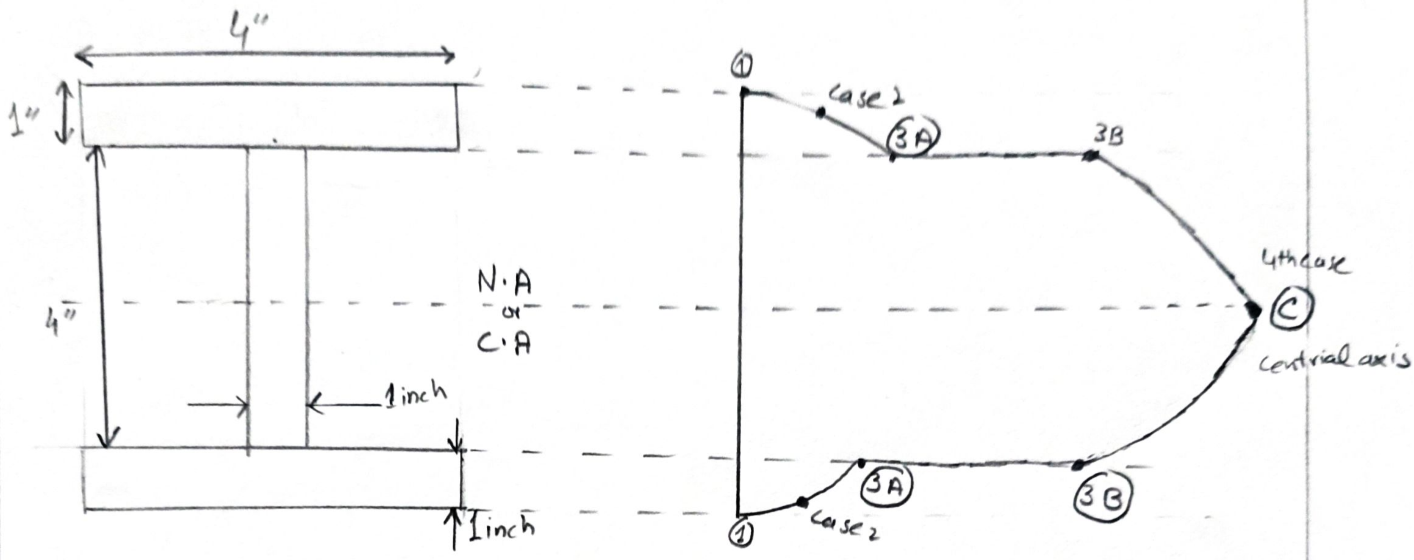
$\boxed{Q = 12 \text{ in}^3}$

$\tau = VQ_{max}/Ib = \frac{144 \times 12}{56 \times 1}$

$\boxed{\tau = 30.85 \text{ psi}}$

⇒ As I section is symmetrical about x-axis and y-axis so same stresses will occur below of centriodal axis as occcured above.

Shear Stress veriation diagram:-



P.T.O

FLEXURAL STRESSES ANALYSIS

For Flexural stress Analysis we consider M_{max} which is.

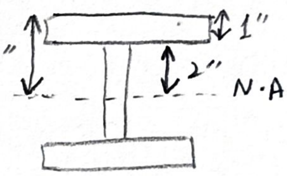
$$M_{max} = 373.24 \text{ lb-ft}$$
$$= 4478.88 \text{ lb-in}$$

$$\sigma = My/I$$

Case: 1

$$\sigma_{Top} = \frac{4478.88 \times 3}{56}$$

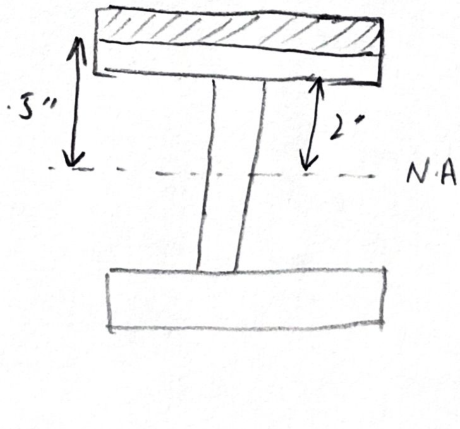
$$\boxed{\sigma_{Top} = 239.94 \text{ lb/in}^2}$$



Case: 2

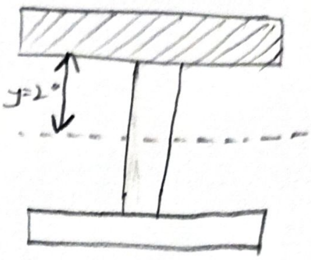
$$\sigma_{0.5} = \frac{4478.88 \times 2.5}{56}$$

$$\boxed{\sigma_{0.5} = 199.95 \text{ psi}}$$



Case: 3

$$\sigma_1 = \frac{4478.88 \times 2}{56}$$

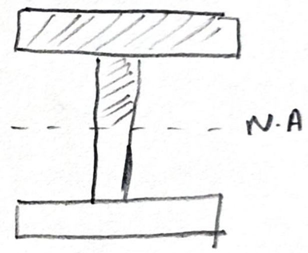


$$\boxed{\sigma_1 = 159.96 \text{ psi}}$$

Case: 4

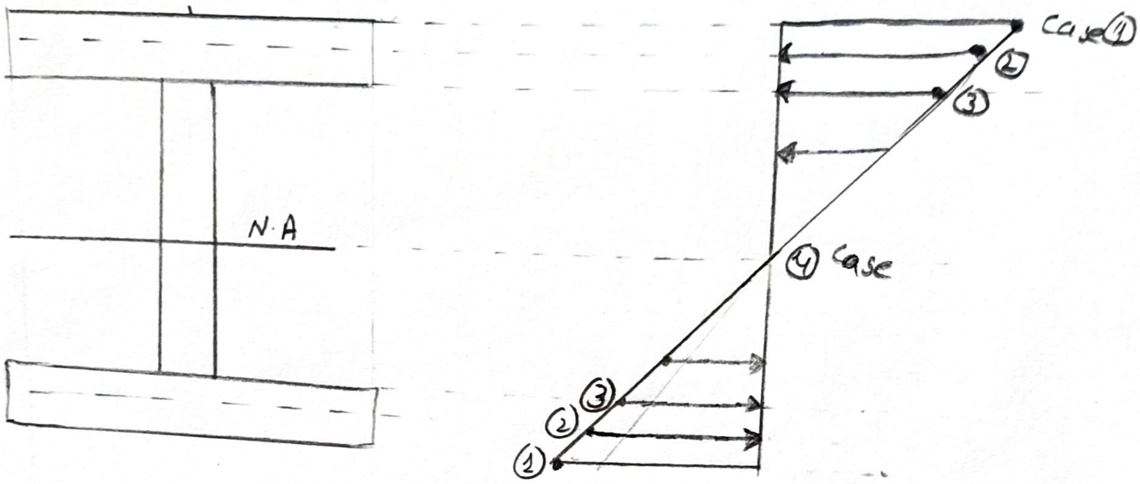
$$\sigma_{N.A} = \frac{4478.88 \times 0}{56}$$

$$\sigma_{N.A} = 0$$



Note: Because of symmetrical structure of "I" Section Flexural stresses below N.A (C.A) will be same as above.

Flexural stresses Distribution diagram:



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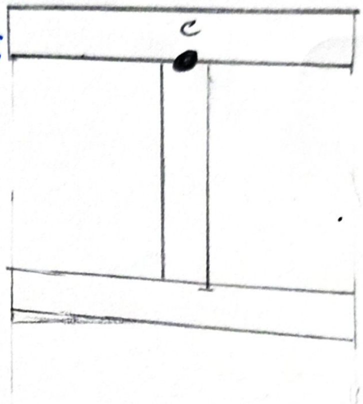
⇒ Stress State of a point element C.

Stresses at poin "C" located at 3ft from left support & 1" below top fiber.

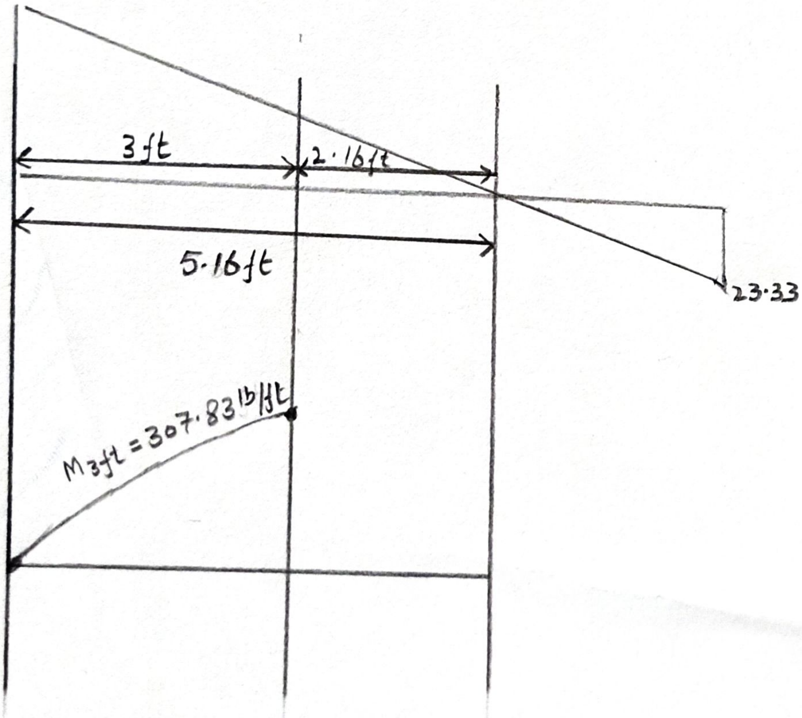
FROM S.F.D RIGHT ANGLE TRINGE



$$\frac{V_{3ft}}{(5.16-3)} = \frac{144.67}{5.16}$$

$$V_{3ft} = \frac{2.16 \times 144.67}{5.16}$$

$$\boxed{V_{3ft} = 60.55\,lb}$$



$$M_{3ft} = (3 \times 60.55) + 3 \times (144.67 - 60.55)$$

$$\boxed{= 307.83\ lb \times ft}$$

$$\boxed{= 3693.96\ lb \times ft}$$

Shear Stress at point C:-

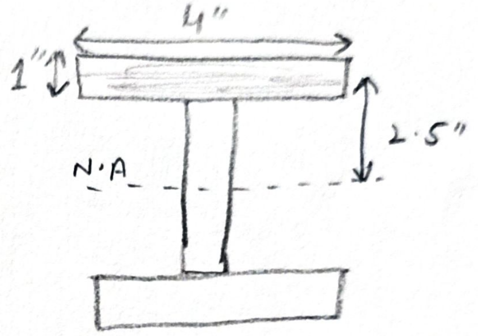
=> As point "C" lies 1 in below top fiber so we will take 2 values; $b = 1''$, $b = 4''$

Condition = $b \Rightarrow 1''$

$b = 1''$

$V = 60.55$

$I = 56\ in^4$



$$\tau_{c3ft} = \frac{VQ}{Ib}$$

$$Q = \bar{y} \times A$$

$$Q = 2.5 \times (4 \times 1)$$

$$\boxed{Q = 10\ in^3}$$

$$\tau_{c3ft} = \frac{60.55 \times 10}{56 \times 1}$$

$$\boxed{\tau_{c3ft} = 10.81\ lb}$$

2nd Condition

Now $b = 4''$

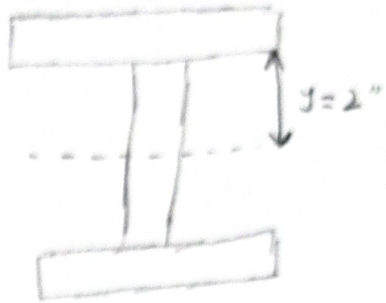
$$\tau_{c3ft} = \frac{60.55 \times 10}{56 \times 1}$$

$$\boxed{\tau_{c3ft} = 2.7\ lb}$$

FLEXURAL STRESS AT POINT C:-

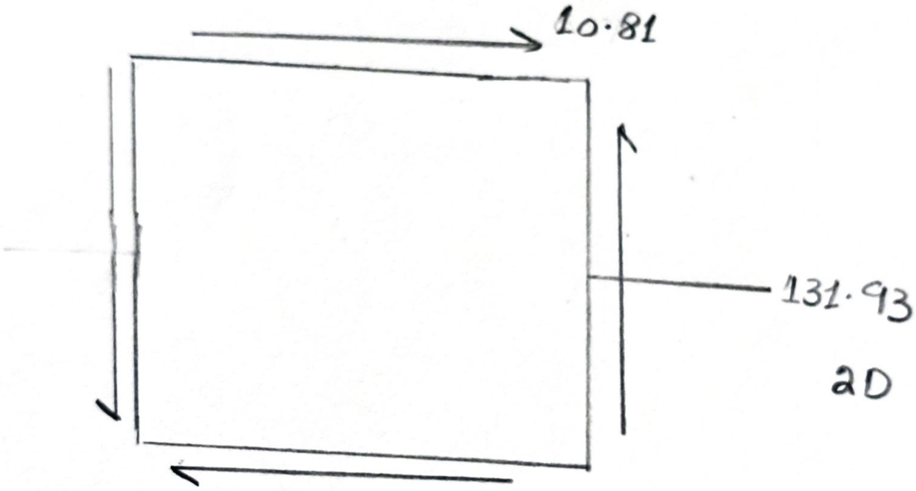
$$\sigma_c = \frac{MY}{I} \Rightarrow \frac{3693.96 \times 2}{56}$$

$$\boxed{\sigma_c = 131.93 \text{ psi}}$$

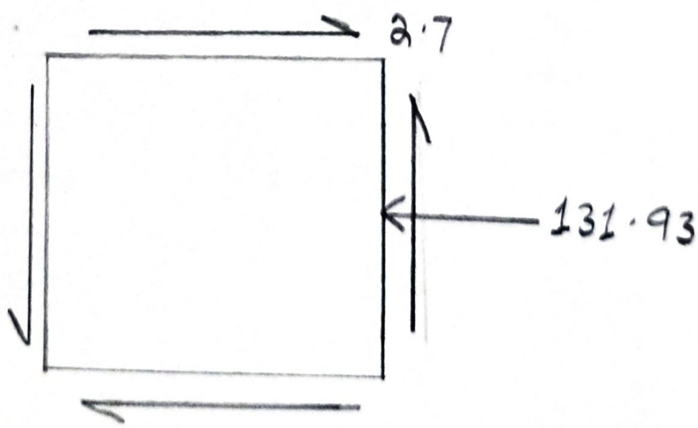


Condition => 01

b = 1



Second Condition => **b = 4"**



"PRINCIPAL STRESSES"

Condition: 01

$$\tau = 10.81\ lb$$

$$\sigma_x = -131.93\ lb$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\tan 2\theta_p = \frac{10.81}{(-131.93-0)/2}$$

$$\tan 2\theta p = 0.164$$

$$2\theta = \tan^{-1}(0.164)$$

$$\theta_p = -9.31/2$$

$$\boxed{\theta_p = -4.65}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-131.93+0}{2} - \sqrt{\left(\frac{-131.93-0}{2}\right)^2 + (10.81)^2}$$

$$\sigma_1 = -65.965 - 66.84$$

$$\boxed{\sigma_1 = -132.805_{psi}}$$

$$\sigma_2 = \frac{-131.93+0}{2} + \sqrt{\left(\frac{-131.93-0}{2}\right)^2 + (10.81)^2}$$

$$\sigma_2 = -65.965 + 66.84$$

$$\boxed{\sigma_2 = 0.875_{psi}}$$

MAX in plane Shear Stress for Condition 1

$\tau_{xy} = 10.81$ psi

$\sigma_x = -131.93$ psi

$$\tan 2\theta_x = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_x = -\frac{(-131.93 - 0)/2}{10.81}$$

$$\tan 2\theta_x = 6.10$$

$$\boxed{\theta_x = 40.34°}$$

$$\tau_{\substack{max\ in \\ plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-131.93 - 0}{2}\right)^2 + (10.81)^2}$$

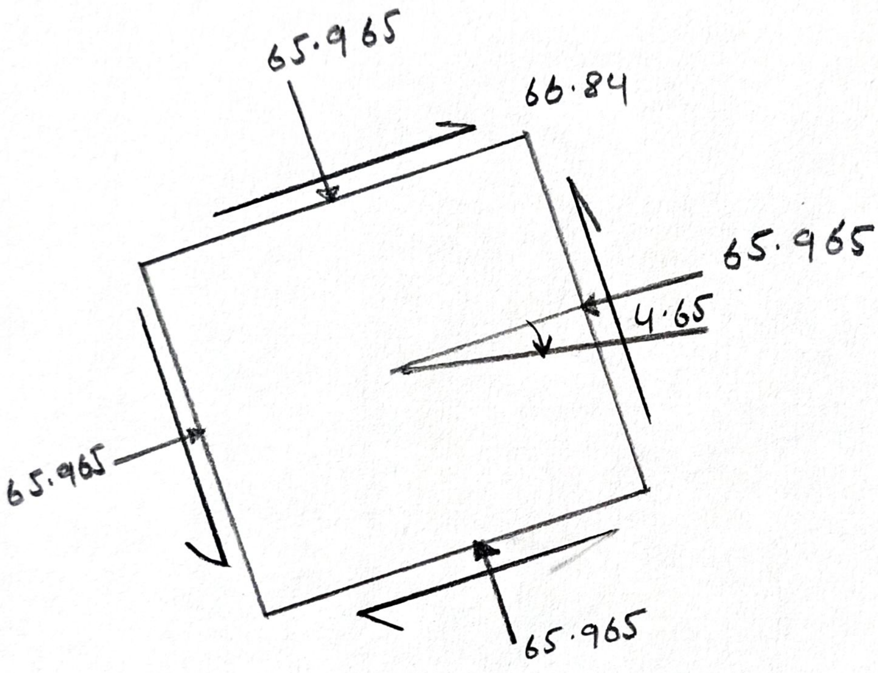
$$\boxed{= 66.84 \text{ psi}}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{avg} = \frac{-132.805 + 0.875}{2}$$

$$\sigma_{avg} = -65.965 \text{ psi}$$

$$\boxed{\theta = -4.65^\circ}$$



⟹ "The difference b/w principal stress & max in plane shear stress is 45° degree."

"MOHR'S CIRCLE"

$\sigma_x = -131.93$ psi $\quad \sigma_y = 0$

$\tau_{xy} = 10.81$ psi

$$R = \sqrt{\frac{-131.93 - 0}{2} + 10.81^2}$$

$$\boxed{R = 66.84 \text{ psi}}$$

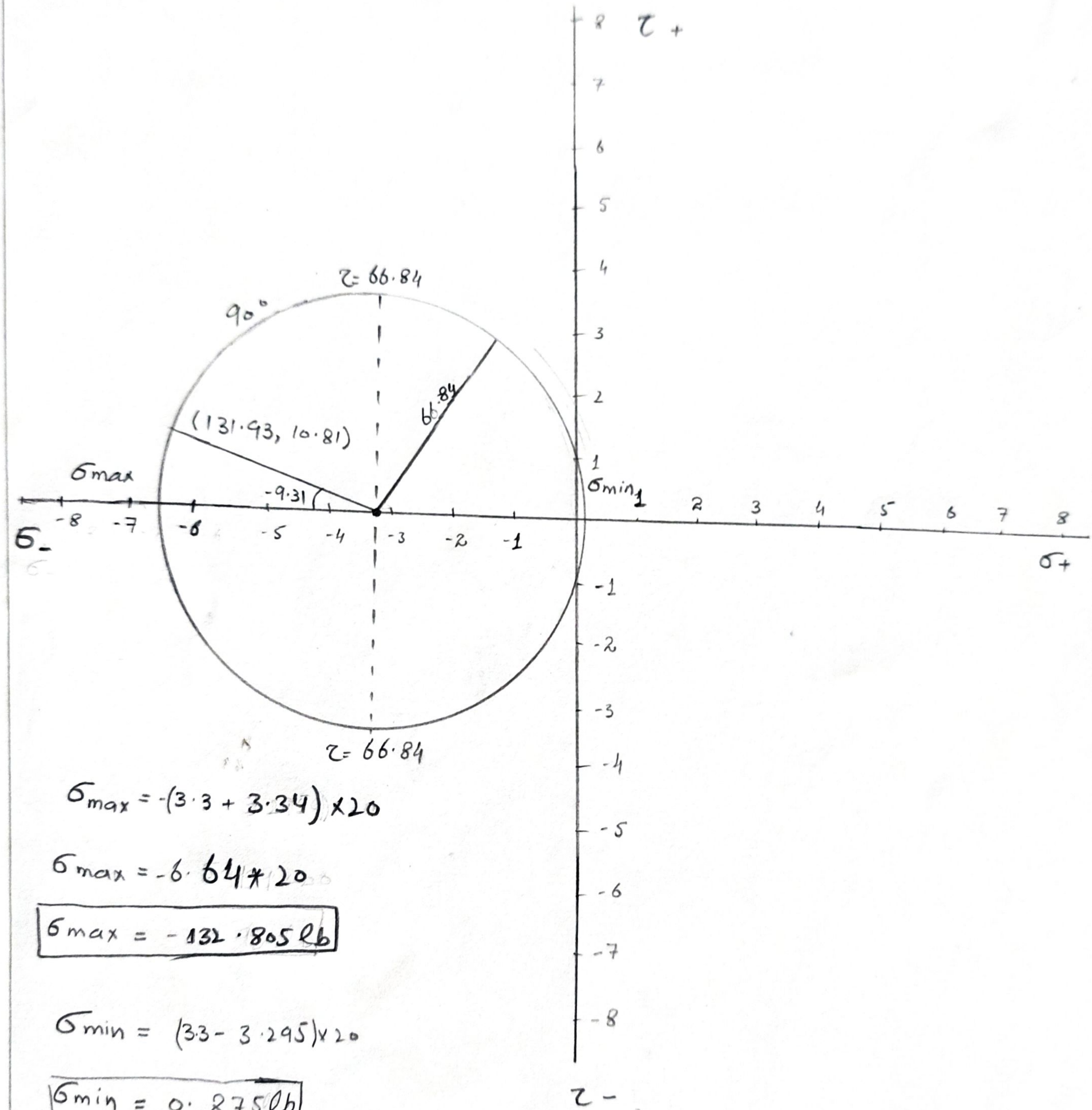
"Center Co-ordinates"

$$(h, k) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$= \left(\frac{-131.93 + 0}{2}, 0\right)$$

$$\boxed{= (-65.956, 0)}$$

SCALE USED: 1cm = 20Psi



$\sigma_{max} = -(3.3 + 3.34) \times 20$

$\sigma_{max} = -6.64 * 20$

$\boxed{\sigma_{max} = -132.805 \, lb}$

$\sigma_{min} = (3.3 - 3.295) \times 20$

$\boxed{\sigma_{min} = 0.875 \, lb}$

PRINCIPAL STRESS CONDITION # 02

FOR $J = 2.7\ lb$

$\sigma_n = -131.93\ lb$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\sigma_x - \sigma_y / 2}$$

$$= \frac{2.7}{(-131.93 - 0)/2}$$

$$\tan 2\theta_p = -0.041$$

$$2\theta_p = \tan^{-1}(0.041)$$

$$\theta_p = -1.173°$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{-131.93 + 0}{2} - \sqrt{\left(\frac{-131.93 - 0}{2}\right)^2 + 2.7^2}$$

$$\sigma_1 = -65.965 \mp 66.62$$

$$\boxed{\sigma_1 = -131.985\ psi}$$

$$\sigma_2 = \frac{-131.93 + 0}{2} + \sqrt{\left(\frac{-131.93 + 0}{2}\right)^2 + 2.7^2}$$

$$\sigma_2 = -65.965 + 66.02$$

$$\boxed{\sigma_2 = 0.055\ psi}$$

MAX IN PLANE SHEAR STRESS FOR CONDITION = 22

$\tau_{xy} = 2.7\ psi$, $\sigma_{xy} = -131.93\ psi$

$$\tan 2\theta_{\cancel{x}} = \frac{-(\sigma_x + \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_x = \frac{-(-131.93 + 0)/2}{2.7}$$

$$\tan 2\theta_x = 24.43$$

$$2\theta_x = \tan^{-1} 24.43$$

$$\boxed{\theta_x = 43.82^\circ}$$

$$\tau_{\substack{max \\ in\ plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

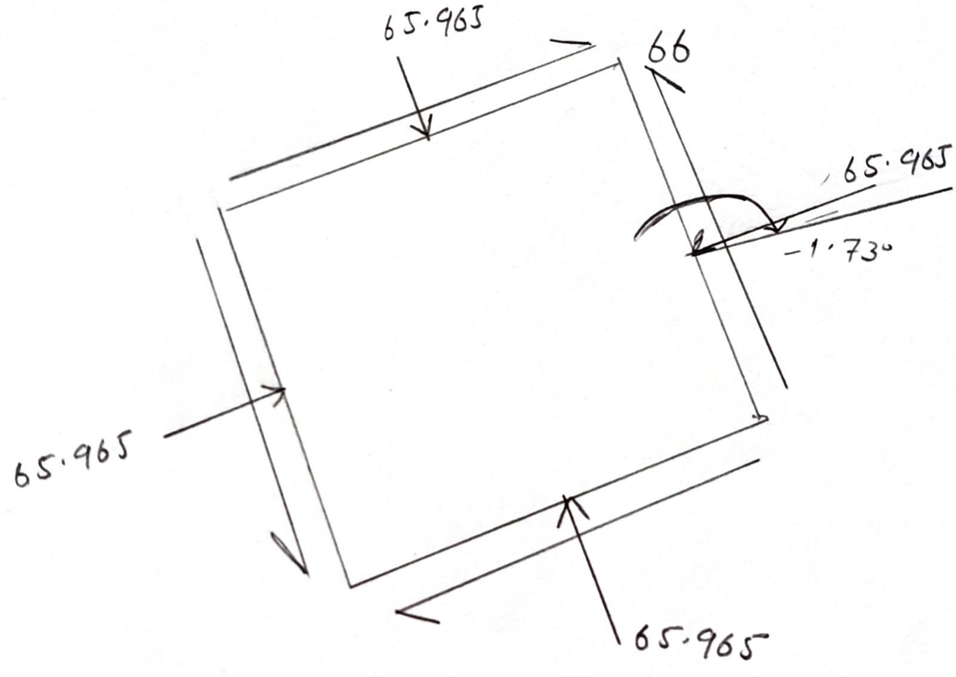
$$= \sqrt{\left(\frac{-131.93 - 0}{2}\right)^2 + (2.7)^2}$$

$$\boxed{= 66\ psi}$$

$$\sigma_{avg} = \frac{\sigma_n + \sigma_y}{2}$$

$$\sigma_{avg} = \frac{-131.985 + 0.055}{2}$$

$$\boxed{\sigma_{avg} = -65.965 \text{ psi}}$$



"MOHR'S CIRCLE"

$\sigma_x = -131.93$ psi , $\sigma_y = 0$

$\sigma_{xy} = 2.7$ psi

$$R = \sqrt{\left(\frac{-131.93-0}{2}\right)^2 + (2.7)^2}$$

$$R = 66 \text{ psi}$$

Center Co-ordinates

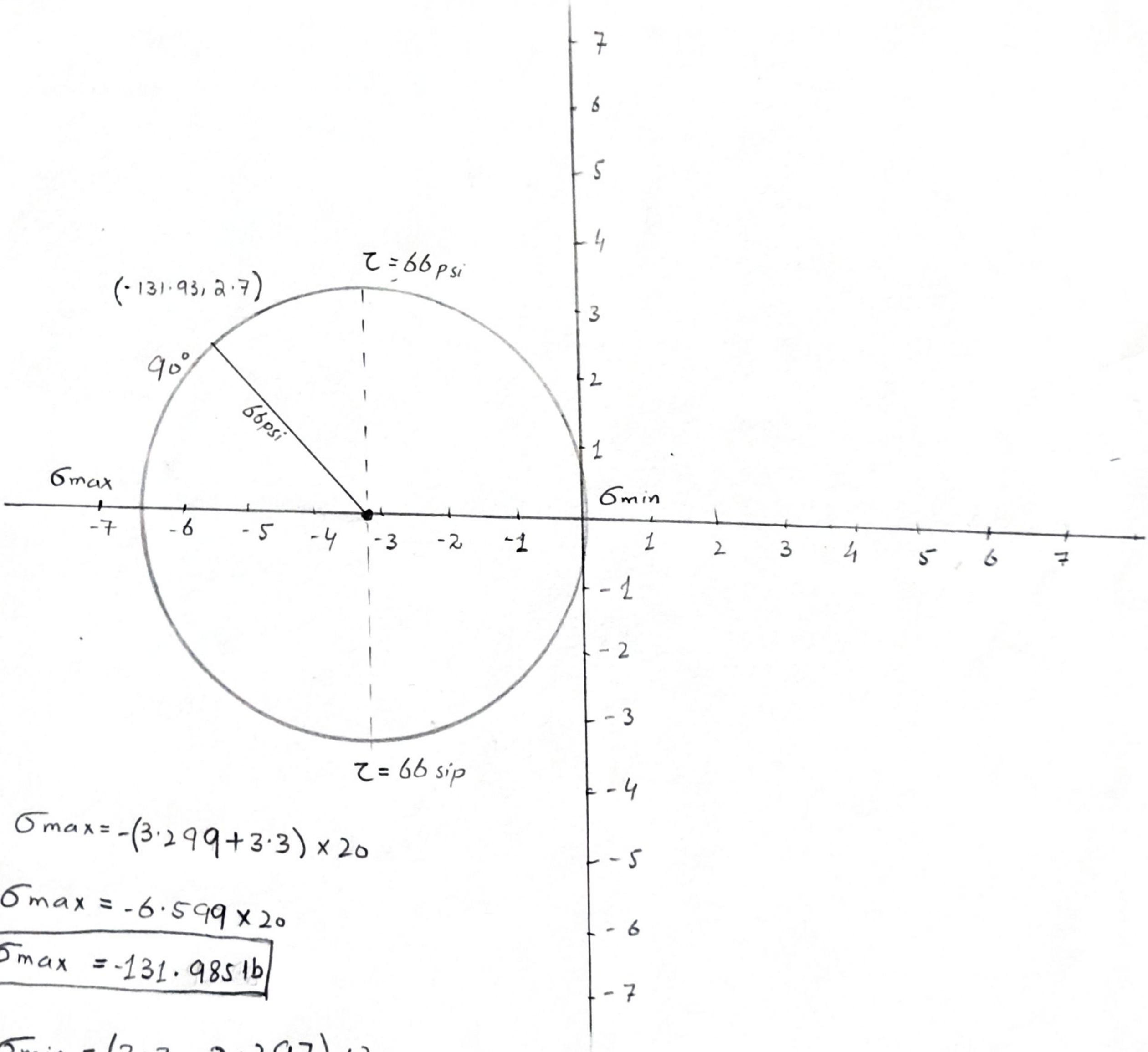
$$(h,k) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$(h,k) = \left(\frac{-131.93+0}{2}\right), 0$$

$$\boxed{(h,k) = (-65.956, 0)}$$

2nd Condition

Scale Used 1cm = 20 psi



$\sigma_{max} = -(3.299 + 3.3) \times 20$

$\sigma_{max} = -6.599 \times 20$

$\boxed{\sigma_{max} = -131.985 \text{ lb}}$

$\sigma_{min} = (3.3 - 3.297) \times 20$

$\sigma_{min} = (3 \times 10^{-3}) \times 20$

$\boxed{\sigma_{min} = 0.055 \text{ lb}}$