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SUBJECT: LINEAR ALGEBRA

DEPARTMENT: BS SOFTWARE ENGG

SECTION: A

SEMESTER: 2nd

Q1: Compute the adjoint of;

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

ID = 15958
∴ 5 is 2nd id number.

Sol: Given: $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

First we find cofactor of every element of A.

$$A_{11} = (-1)^{1+1} \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= 1 \cdot (6-1)$$

$$\boxed{A_{11} = 5}$$

$$A_{12} = (-1)^{1+2} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$= -1\ (4-3)$$

$$\boxed{A_{12} = -1}$$

$$A_{13} = (-1)^{1+3} \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= 1 \; (2 - 9)$$

$$\boxed{A_{13} = -7}$$

$$A_{21} = (-1)^{2+1} \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$$

$$= -1 \; (4 - 5)$$

$$\boxed{A_{21} = 1}$$

$$A_{22} = (-1)^{2+2} \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}$$

$$= 1 \; (2 - 15)$$

$$\boxed{A_{22} = -13}$$

$$A_{23} = (-1)^{2+3} \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$$

$$= -1 \; (1 - 6)$$

$$\boxed{A_{23} = 5}$$

$$A_{31} = (-1)^{3+1} \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$= 1 \; (2 - 15)$$

$$\boxed{A_{31} = -13}$$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix}$

$= -1 \ (1-10)$

$\boxed{A_{32} = +9}$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

$= 1 \ (3-4)$

$\boxed{A_{33} = -1}$

So $\operatorname{adj} A = \begin{bmatrix} 5 & -1 & -7 \\ 1 & -13 & 5 \\ -13 & 9 & -1 \end{bmatrix}$

Now taking the transpose

$\operatorname{Adj} A = \begin{bmatrix} 5 & 1 & -13 \\ -1 & -13 & 9 \\ -7 & 5 & -1 \end{bmatrix}$

Answer

ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

Sol:

Cofactor of B are;

$B_{11} = (-1)^{1+1} \begin{bmatrix} -1 & 8 \\ -2 & 8 \end{bmatrix}$

Mij

$= 1 \, (-8 + 16)$

$\boxed{B_{11} = 8}$

$B_{12} = (-1)^{1+2} \begin{bmatrix} 2 & 8 \\ 5 & 8 \end{bmatrix}$

$= -1 \, (16 - 40)$

$\boxed{B_{12} = 24}$

$B_{13} = (-1)^{1+3} \begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

$= 1 \, (-4 + 5)$

$\boxed{B_{13} = 1}$

$$B_{21} = (-1)^{2+1} \begin{bmatrix} 4 & 5 \\ -2 & 8 \end{bmatrix}$$

$$= -1\ (32 + 10)$$

$$\boxed{B_{21} = -42}$$

$$B_{22} = (-1)^{2+2} \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix}$$

$$= 1\ (24 - 25)$$

$$\boxed{B_{22} = -1}$$

$$B_{23} = (-1)^{2+3} \begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix}$$

$$= -1\ (-6 + 20)$$

$$\boxed{B_{23} = -14}$$

$$B_{31} = (-1)^{3+1} \begin{bmatrix} 4 & 5 \\ -1 & 8 \end{bmatrix}$$

$$= 1\ (32 + 5)$$

$$\boxed{B_{31} = 37}$$

$$B_{32} = (-1)^{3+2} \begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix}$$

$$= -1\ (24 - 10)$$

$$B_{32} = -14$$

$$B_{33} = (-1)^{3+3} \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$$

$$= 1\ (-3 - 8)$$

$$B_{33} = -11$$

So $\operatorname{adj} B = \begin{bmatrix} 8 & +24 & 1 \\ -42 & -1 & -14 \\ 37 & -14 & -11 \end{bmatrix}$

Now taking the transpose.

$$\operatorname{Adj} B = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & -14 & -11 \end{bmatrix}$$

Answer.

CP 2: Find the co-factor of A_{21}, A_{31}, A_{33} if;

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Solution: For A_{21}

As we know that

cofactors of $a_{ij} = A_{ij} = (-1)^{i+j} M_{ij}$

So:

$$a_{21} = A_{21} = (-1)^{2+1} M_{21} \quad \text{—} \quad (i)$$

$$M_{21} = \begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix} = -4-(-9)$$

$$\boxed{M_{21} = 5}$$

$$(i) \Rightarrow A_{21} = (-1)^{3}(5)$$

$$\boxed{A_{21} = -5}$$

P-T-O

For A_{31}

Similarly.

$$a_{31} = A_{31} = (-1)^{3+1} M_{31} \quad \text{—— (ii)}$$

$$M_{31} = \begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix} = -2 - 9$$

$$\boxed{M_{31} = -11}$$

(ii) $\Rightarrow A_{31} = (-1)^{3+1} (-11)$

$$\boxed{A_{31} = -11}$$

For A_{33}

$$a_{33} = A_{33} = (-1)^{3+3} M_{33} \quad \text{—— (iii)}$$

$$M_{33} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} = 3 + 4$$

$$\boxed{M_{33} = 7}$$

(iii) $\Rightarrow A_{33} = (-1)^{6} (7)$

$$\boxed{A_{33} = 7}$$

Answer

Q3: Find Eigen values and eigen vector if;

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Sol:

∵ Let λ be the eigen value and $\vec{x} = (x_1, x_2, x_3)$ be the corresponding eigen vector then to find λ we from.

$$A - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\Rightarrow A - \lambda I = \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix}$$

P.T.O

Next

det $(A-\lambda I)=0$ gives the values of λ.

$$\det(A-\lambda I)=0$$

$$\det\begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix}=0$$

$$\Rightarrow (2-\lambda)\left[(3-\lambda)(2-\lambda)-2(1)\right]-1\left[1(2-\lambda)-2(-1)\right] +1\left[1(1)-(3-\lambda)(-1)\right]=0$$

$$\Rightarrow (2-\lambda)\left[6-3\lambda-2\lambda+\lambda^2-2\right]-1\left[2-\lambda+2\right]+1\left[1+3-\lambda\right]=0$$

$$\Rightarrow (2-\lambda)\left[\lambda^2-5\lambda+4\right]-\cancel{4}+\cancel{\lambda}+\cancel{4}-\cancel{\lambda}=0$$

$$\Rightarrow 2\lambda^2-10\lambda+8-\lambda^3+5\lambda^2-4\lambda=0$$

$$\Rightarrow -\lambda^3+7\lambda^2-14\lambda+8=0$$

$$\Rightarrow \lambda^3-7\lambda^2+14\lambda-8=0$$

$$\Rightarrow (\lambda-2)(\lambda^2-5\lambda+4)=0$$

P-9-0

$(\lambda-2)=0$ OR $\lambda^2-5\lambda+4=0$

$\lambda=2$ OR $\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2ac}$

$\lambda = 1, 4.$

Eigen values are;

$\lambda = 1, 2, 4$

Now eigen vector w.r.t $\lambda=1$ are given by;

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 1 & 3-1 & 2 \\ -1 & 1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_1 & 2x_2 & 2x_3 \\ -x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

P-T-O

$$x_1 + x_2 + x_3 = 0 \quad \text{— (i)}$$
$$x_1 + 2x_2 + 2x_3 = 0 \quad \text{— (ii)}$$
$$-x_1 + x_2 + x_3 = 0 \quad \text{— (iii)}$$

Adding (i) and (iii)

$$\cancel{x_1} + x_2 + x_3 = 0$$
$$-\cancel{x_1} + x_2 + x_3 = 0$$
$$2x_2 + 2x_3 = 0 \quad \text{— (4)}$$

Subtract (4) from (2)

$$x_1 + \cancel{2x_2} + \cancel{2x_3} = 0$$
$$+\cancel{2x_2} + \cancel{2x_3} = 0$$
$$x_1 = 0$$

$$E_1 = (0, 1, -1)$$
$$E_2 = (0, 2, -2)$$

P-7-0

Eigen vectors w.r.t $\lambda = 2$ are given by

$$\begin{bmatrix} 2-2 & 1 & 1 \\ 1 & 3-2 & 2 \\ -1 & 1 & 2-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & x_2 & x_3 \\ x_1 & x_2 & 2x_3 \\ -x_1 & x_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_2 + x_3 = 0$ — ①

$x_1 + x_2 + 2x_3 = 0$ — ②

$-x_1 + x_2 = 0$ — ③

subtract ① from ②

$$\begin{aligned} x_1 + \cancel{x_2} + 2\cancel{x_3} &= 0 \\ \cancel{x_2} \pm \cancel{x_3} &= 0 \\ \hline x_1 + x_3 &= 0 \end{aligned}$$ — ④

$E_1 = (1, 1, -1)$

$E_2 = (2, 2, -2)$

P-T-O

Eigen vectors w.r.t $\lambda = 4$ are given by,

$$\begin{bmatrix} 2-4 & 1 & 1 \\ 1 & 3-4 & 2 \\ -1 & 1 & 2-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{R} \begin{bmatrix} -2 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + x_3 = 0$$

So;

$$E_1 = (1, 1, -1)$$

$$E_2 = (2, 2, -2)$$

Answer