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Subject : DSP.

Paper : Mid term

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(1)

Q#1

following analog signal.

$$x_a(t) = 3 \cos 100\pi t + 4 \sin 200\pi t.$$

Soln:

(i)

According to Sampling theorem,

$$f_1 = 100 \text{ Hz} \quad f_2 = 200 \text{ Hz}.$$

$$f_s \geq f_{\max}.$$

$$\therefore f = \omega / 2\pi.$$

So,

f_2 is maximum then f_1 .

$$f_s > 2 \times 100.$$

$$f_s = 200 \text{ Hz}$$

(ii)

Suppose s-rate $F_s = 100 \text{ Hz}$
discrete-time signal.

effect of sampling rate on
newly generated d. time signal.

Soln:

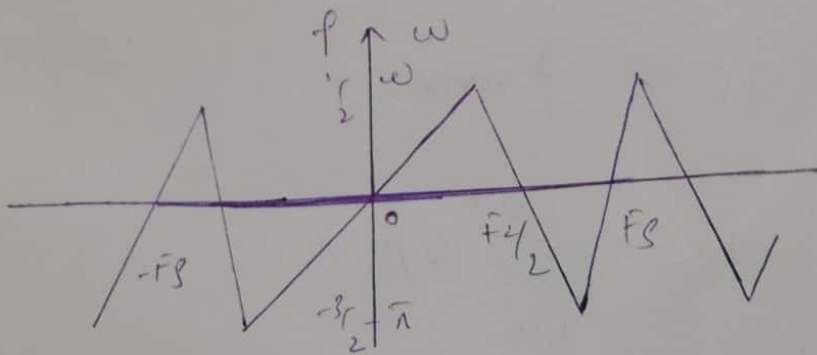
(2)

$$f_s = 100 \text{ Hz.}$$

$$f = 100/2 \Rightarrow 50 \text{ Hz.}$$

$$x_a[n] = 3 \cos \pi \left(\frac{50}{100} \right) n + 4 \sin 2\pi \left(\frac{100}{100} \right) n.$$

$$= 3 \cos \pi \left(\frac{1}{20} \right) n + 4 \sin 2\pi n.$$



\therefore The new generated discrete time signal is that. There will be no display phenomena mean, There will be not present unwanted component in Reconstruct of the signal.

(iii) what is analog signal $y_a(t)$ we can construct from sample if we use ideal interpolation. (3)

Ans

foldling frequency = $f_1/2 = \frac{100}{2} = 50\text{Hz}$.
 $f_1 = 50\text{Hz}$, $f_2 = 100\text{Hz}$

original signal $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$.

Since only the frequency components at 100Hz are present on the sampled signal the analog signal we can recover or reconstruct

y $y_a(t) = 3\cos 100\pi t$ Ans.

Q1 (D) Consider discrete t. signal. (4)

$$x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

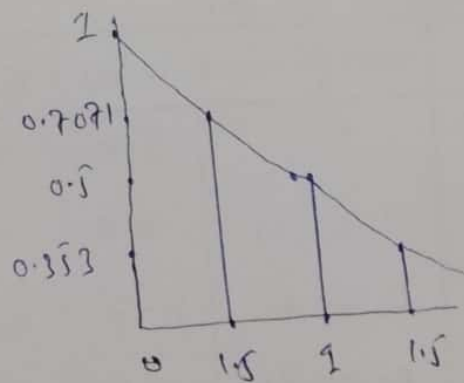
signal sampled rate is $f_s = 2 \text{ Hz}$.

(2) Draw sampled signal.

$$f_s = \frac{1}{T} = T = \frac{1}{f_s}$$

$$T = 0.5 \text{ sec.}$$

x_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



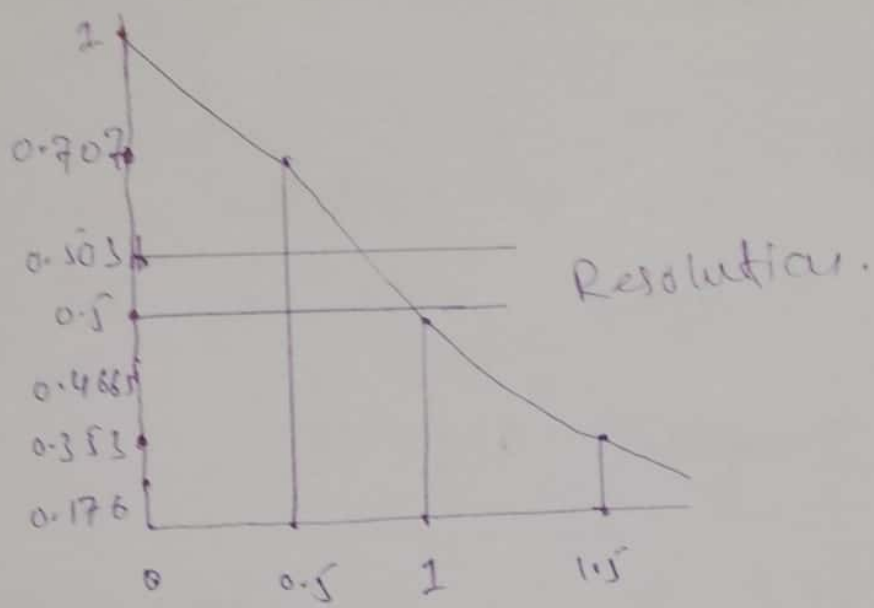
iii

Ans

$$L = 2^n, \quad n = \text{bits} = 3$$

$$L = 2^3 = 8 \text{ levels.}$$

$$\begin{aligned} \text{Resolution} &= \frac{x_{\max} - x_{\min}}{L} \\ &= \frac{1 - 0}{8} \Rightarrow 0.125 \end{aligned}$$



iii)

	Discrete signal	Input	Reply	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.2	0.2	-0.1

Q112 Determine response
of the signal.

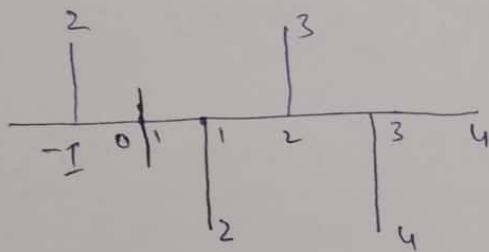
(6)

(a) $x[n] = \{ 2, \underset{\uparrow}{1}, -2, 3, -4 \}$

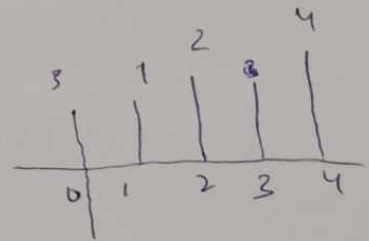
$h[n] = \{ \underset{\uparrow}{3}, 1, 2, 1, 4 \}$

Soln:

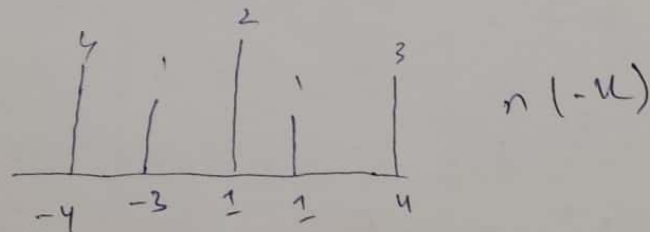
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$



www



$n(-k)$ = folded signal

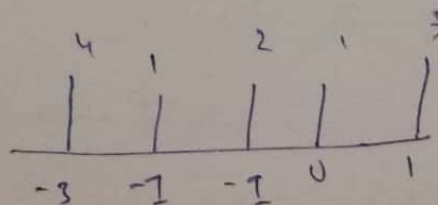


$$y[0] = \sum_{k=-4}^0 n[-k] h[-k] + 1(0) (h(0))$$

$$= 2 + 3 \Rightarrow 5$$

for $n=1$.

$n(1-k)$



$$\begin{aligned}
 y[2] &= \sum_{k=1}^0 n[k] (n[1-k]) \quad (7) \\
 &= n[-2]n[-2] + n[0]n[0] + n[1]n[1] \\
 &= 4 + 9 - 6 \Rightarrow -1
 \end{aligned}$$

∴ $n=2$



$n[2-k]$

$$\begin{aligned}
 y[2] &= \sum_{k=2}^2 n[k] n[2-k] \\
 &= n[-2]n[-2] + 2n[0] + n[0] + n[2]n[2] \\
 &= 2 + 2 - 2 + 9 \\
 &= 11
 \end{aligned}$$

As $n=3$

$$y[3] = -8$$

$n=4$

$$y[4] = 4$$

$n=5$

$$y[5] = 13$$

$n=6$

$$y[6] = 8$$

(b) Compute the convolution $y[n]$ of the following signal.

$$x[n] = \begin{cases} a^{n-2} & , -3 \leq n \leq 5 \\ 0 & , \text{else where.} \end{cases}$$

$$h[n] = \begin{cases} 2^n & 0 \leq n \leq 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Solu:

we have.

$$x[n] = x[k] = \{ a^{-2}, a^{-1}, 1, a, a^2, a^3, \dots, a^6, 0, a, \dots \}$$

$$h[n] = h[k] = \{ \dots, 0, 1, 2, 4, 8, 16, 0, \dots \}$$

To find $y[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

for $n=0$ first to find $h[n-k] = h[0-k]$
by inverting $h[k]$ we get $h[-k]$.

$$\Rightarrow h[-k] = \{ 16, 8, 4, 2, \frac{1}{2} \} \quad \text{--- (4)}$$

$$\text{So } y(0) = \sum_{k=-\infty}^{\infty} x[k] \times h[-k]$$

$$y(0) = (a^{-2} + 4a^{-3} + 2 + 2) \\ = a^{-2} + 4a^{-2} + 4a^{-2}$$

for $n=1$.

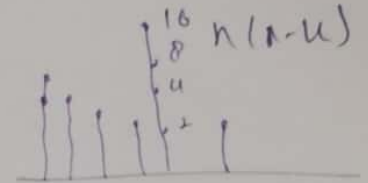
(9)

$$h(1-u) = \{16, 8, 4, 2, 1\}$$

so

$$y(1) = (\alpha^{-2} \times 16) + (\alpha^{-2} \times 8) + (1 \times 4) + (\alpha \times 2) + (\alpha^2 \times 1)$$

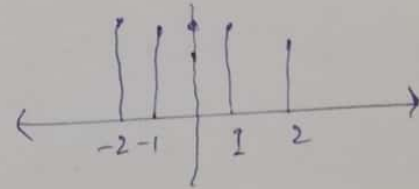
$$\begin{aligned} y(1) &= (16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^2) \\ &= \alpha^2 + 2\alpha + 4 + 4 + 8\alpha^{-1} + 16\alpha^{-2} \end{aligned}$$



Now

for $n=2$.

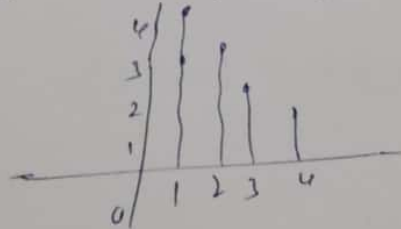
$$y(2) = 16\alpha^{-2} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$$



Similarly

for $n=3$

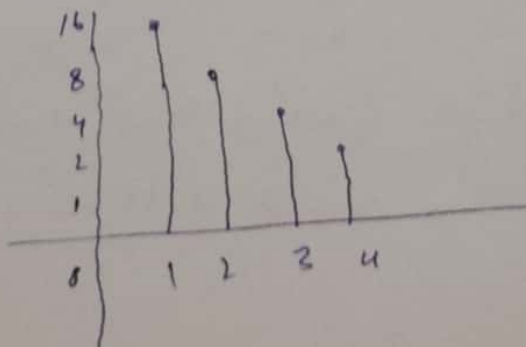
$$y(3) = 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$$



Now

$$h(4-u) = \{16, 8, 4, 2, 1\}$$

$$\begin{aligned} y(4) &= (\alpha^1 \times 16) + (\alpha^2 \times 8) + (\alpha^3 \times 4) + (\alpha^4 \times 2) + (\alpha^5 \times 1) \\ &= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5 \end{aligned}$$



Similarly $x(5-n) = y(n)$ 10

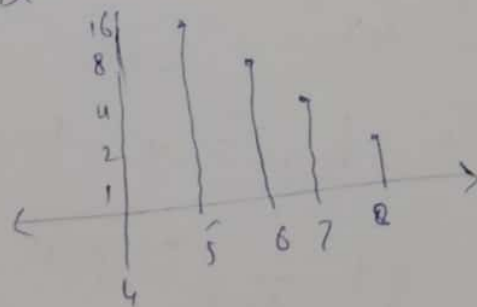
$$\therefore y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$$

$$\therefore y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

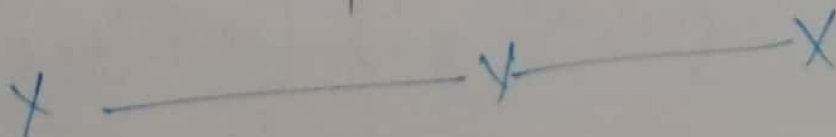
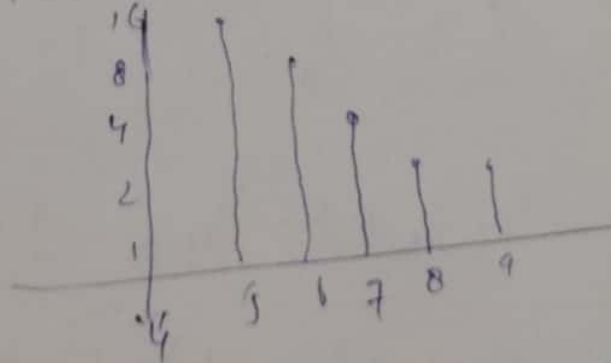
$$\therefore y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$

and there discrete time signal saw $y[4], \dots$

$$y[8] = 16\alpha^5 + 8\alpha^6$$



$$y[9] = 0 + 0 + 0 + 0 + 0 + 16\alpha^6$$
$$= 16\alpha^6$$



Q#8 Determine z-transform. (11)

$$(7) \quad x(n) = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0 \end{cases}$$

Solu:

As we know that z-transform.

$$X(z) = \sum_{n=0}^{\infty} (1/4)^n z^{-n} + \sum_{n=-\infty}^0 (1/3)^{-n} z^{-n} - 1.$$

using geometric series.

$$\Rightarrow \frac{1}{1 - \frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^{-n} - 1.$$

$$\Rightarrow \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - \frac{1}{3}z^{-1}} - 1.$$

$$\Rightarrow \frac{1 - \frac{1}{4}z^{-1} + 1 - \frac{1}{3}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})}.$$

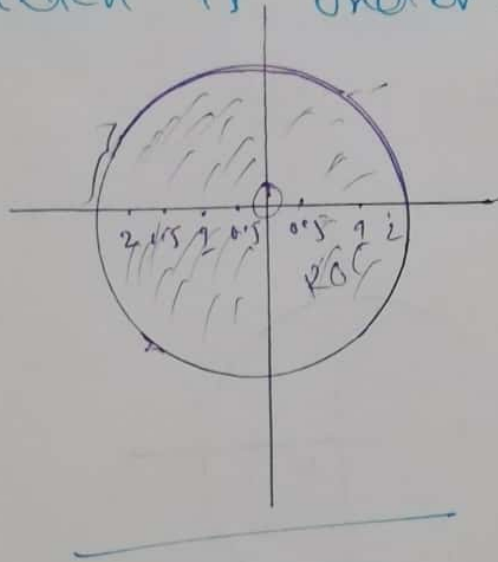
$$\Rightarrow \frac{1 - \frac{1}{3}z^{-1} + 1 - \frac{1}{4}z^{-1}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

$$\Rightarrow \frac{1 - \frac{1}{3}z^{-1} + 1 - \frac{1}{4}z^{-1} - \frac{1}{3}z^{-1} + \frac{1}{4}z^{-1} + \frac{1}{12}z^{-2} + \frac{1}{12}z^{-2}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z^{-1})}$$

$$\begin{aligned}
 &= \frac{1 - \frac{1}{12}}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)} \quad (b) \\
 &= \frac{13/12}{\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}
 \end{aligned}$$

Hence The ROC is $\frac{1}{4} < |z| < 3$

The sketch is under.



(ii) $x(n) = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$

Soln
z-transform.

$x(n) = \alpha^4 h[n] \quad \text{---} \quad h[z] = \frac{1}{1 - \alpha z^{-1}} \quad (B)$

By putting values.

$$x_2(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) 2^{n-n} - \sum_{n=0}^{\infty} 2^4 2^{-n}$$

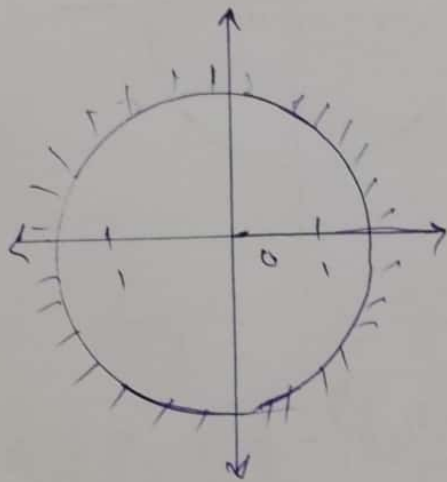
(13)

$$= \frac{1}{1 - \frac{1}{2}z^{-3}} - \frac{1}{1 - 2z^{-1}}$$

$$= \frac{-\frac{5}{2}z^{-1}}{\left(1 - \frac{1}{2}z^{-3}\right)\left(1 - 2z^{-1}\right)}$$

\therefore A seen ROC use $|z| > 2$.

\Rightarrow The sketch are:



THE END.