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Q1: (a) Find the general solution of

$$y' = \frac{(4x^2+y^2)}{xy}$$

Solution:

$$y' = \frac{(4x^2+y^2)}{xy}$$

Let

$$y = ux$$

$$\frac{du}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{4x^2+u^2}{4x^2}$$

$$u + x\frac{du}{dx} = \cancel{x^2}\left[\frac{4+u^2}{\cancel{4x^2}}\right]$$

$$u + x\frac{du}{dx} = \frac{4+u^2}{u}$$

$$x\frac{du}{dx} = \frac{4+u^2}{u} - u$$

$$x\frac{du}{dx} = \frac{4+\cancel{u^2}-\cancel{u^2}}{u}$$

$$x\frac{du}{dx} = \frac{4}{x}$$

$$x \cdot du = 4 \cdot \frac{1}{u} \cdot dx$$

$$u \cdot du = \frac{4}{x}\, dx$$

$$\int u \cdot du = 4\int \frac{1}{x}\, dx$$

$$\frac{u^2}{2} = 4\ln x + c$$

$$u^2 = 8\ln x + 2c$$

$$u^2 = 8\ln x + c_1$$

Since

$$ux = y$$

$$u = y/x$$

$$\Rightarrow \frac{y^2}{x^2} = 8\ln x + c_1$$

$$\boxed{\frac{y^2}{x^2} = 8\ln x + c_1}$$ Ans

Q2: (a) Find the general solution of

$-\pi \sin\pi x \sinh y\, dx + \cos\pi x \cosh y\, dy = 0$

Solution:- $M dx + N dy = 0$

$M = -\pi \sin\pi x \sinh y, \quad N = \cos\pi x \cosh y$

$$\frac{\partial M}{\partial y} = -\pi \sin\pi x \cosh y, \quad \frac{\partial N}{\partial x} = -\pi \sin\pi x \cosh y$$

So, $$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$u \int M dx + k(y)$

$u = \int -\pi \sin \pi x \sinh y \, dx + k(y)$

$= -\pi \left(-\frac{\cos \pi x}{\pi} \right) \sinh y + k(y)$

$u = \frac{\cancel{\pi} \cos \pi x}{\cancel{\pi}} \sinh y + k(y)$

$u = \cos \pi x \sinh y + k(y)$

Now

$$\frac{\partial u}{\partial y} = \cos \pi x \cosh y + \frac{d}{dy} k(y)$$

But

$$\frac{\partial u}{\partial y} = N = \cos \pi x \cosh y$$

So,

$$\cancel{\cos \pi x \cosh y} = \cancel{\cos \pi x \cosh y} + \frac{d}{dy} k(y)$$

$$\frac{d}{dy} k(y) = 0$$

Now

$$\int d k(y) = \int 0 \, dy$$

$k(y) = c_1$ put in eq 1

④

$$u = \cos\pi x \sinh y + c_1$$
$$c_2 = \cos\pi x \sinh y + c_1$$
$$\cos\pi x \sinh y = c_2 - c_1 = c$$

$$\boxed{\cos\pi x \sinh y = c}$$ Ans

Q3. ⓐ Find the general solution of

$$y' = 4y + x$$

Solution:

$$\frac{dy}{dx} - 4y = x$$

$$\frac{dy}{dx} + P(x)y = Q(x) \longrightarrow \text{linear equation}$$

$$\frac{dy}{dx} + (-4)y = x \longrightarrow ①$$

$$I.f = e^{\int -4\,dx} = e^{-4x}$$

xing equation ① with by I.F

$$e^{-4x}\frac{dy}{dx} - 4e^{-4x}y = xe^{-4x}$$

$$\frac{d}{dx}(e^{-4x}.y) = xe^{-4x}$$

$$\int \frac{d}{dx}(e^{-4x}.y)\,dx = \int xe^{-4x}\,dx$$

$$e^{-4x}.y = x\int e^{-4x}dx - \left[\frac{d}{dx}(x)\int e^{-4x}dx\right]dx$$

$$e^{-4x} \cdot y = \frac{xe^{-4x}}{4} + \int \frac{e^{-4x}}{4} dx$$

$$e^{-4x} \cdot y = \frac{-xe^{-4x}}{4} + \frac{1}{4} \int e^{-4x} dx$$

$$= \frac{-xe^{-4x}}{4} - \frac{1}{16} e^{-4x}$$

$$e^{-4x} \cdot y = e^{-4x} \left(\frac{-x}{4} - \frac{1}{16} \right)$$

$$y = e^{4x} \cdot e^{-4x} \left(\frac{-x}{4} - \frac{1}{16} \right)$$

$$\boxed{y = -\left(\frac{x}{4} + \frac{1}{16} \right) + c}$$

Ans