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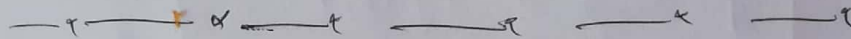
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Dept : MS (Electrical Engineering)

Submitted To : Sir, Hidayatullah

Subject : Advance Engineering Mathematics

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Q1: (a)

Find the general solution of

$$y' = \frac{(4x^2 + y^2)}{xy}$$

Solution:

$$y' = \frac{(4x^2 + y^2)}{xy}$$

Let

$$y = ux$$

$$\frac{du}{dx} = u + x \frac{du}{dx}$$

$$u + x \frac{du}{dx} = \frac{4x^2 + u^2}{4x^2}$$

$$u + x \frac{du}{dx} = \cancel{x^2} \left[\frac{4 + u^2}{\cancel{4x^2}} \right]$$

$$u + x \frac{du}{dx} = \frac{4 + u^2}{u}$$

$$x \frac{du}{dx} = \frac{4 + u^2}{u} - u$$

$$x \frac{du}{dx} = \frac{4 + u^2 - u^2}{u}$$

$$x \frac{du}{dx} = \frac{4}{u}$$

$$u \cdot du = 4 \cdot \frac{1}{u} \cdot dx$$

$$u \cdot du = \frac{4}{x} dx$$

$$\int u \cdot du = 4 \int \frac{1}{x} dx$$

(2)

$$\frac{u^2}{2} = 4 \ln x + c$$

$$u^2 = 8 \ln x + 2c$$

$$u^2 = 8 \ln x + c_1$$

Since

$$ux = y$$

$$u = \frac{y}{x}$$

$$\Rightarrow \frac{y^2}{x^2} = 8 \ln x + c_1$$

$$\boxed{\frac{y^2}{x^2} = 8 \ln x + c_1} \quad \underline{\underline{\text{Ans}}}$$

Q2: @ Find the general solution of
$$-\pi \sin \pi x \sinh y dx + \cos \pi x \cosh y dy = 0$$

Solution:- $M dx + N dy = 0$

$$M = -\pi \sin \pi x \sinh y, \quad N = \cos \pi x \cosh y$$

$$\frac{\partial M}{\partial y} = -\pi \sin \pi x \cosh y, \quad \frac{\partial N}{\partial x} = -\pi \sin \pi x \cosh y$$

$$\text{So, } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

(3)

$$u \int M dx + K(y)$$
$$u = \int -\pi \sin \pi x \sinh y dx + K(y)$$
$$= -\pi \left(-\frac{\cos \pi x}{\pi} \right) \sinh y + K(y)$$

$$u = \frac{\pi \cos \pi x}{\pi} \sinh y + K(y)$$

$$u = \cos \pi x \sinh y + K(y)$$

Now

$$\frac{\partial u}{\partial y} = \cos \pi x \cosh y + \frac{d}{dy} K(y)$$

But

$$\frac{\partial u}{\partial y} = N = \cos \pi x \cosh y$$

So,

$$\cancel{\cos \pi x} \cosh y = \cancel{\cos \pi x} \cosh y + \frac{d}{dy} K(y)$$

$$\frac{d}{dy} K(y) = 0$$

Now

$$\int dK(y) = \int 0 dy$$

$$K(y) = C_1 \text{ put in eq (1)}$$

(4)

$$u = \cos \pi x \sin \pi y + C_1$$

$$C_2 = \cos \pi x \sin \pi y + C_1$$

$$\cos \pi x \sin \pi y = C_2 - C_1 = c$$

$$\boxed{\cos \pi x \sin \pi y = c} \quad \text{Ans}$$

Q3. (a)

Find the general solution of

$$y' = 4y + x$$

Solution:

$$\frac{dy}{dx} - 4y = x$$

$$\frac{dy}{dx} + P(x)y = Q(x) \rightarrow \text{linear equation}$$

$$\frac{dy}{dx} + (-4)y = x \rightarrow (1)$$

$$I.f = e^{\int -4 dx} = e^{-4x}$$

Multiplying equation (1) with I.F

$$e^{-4x} \frac{dy}{dx} - 4e^{-4x} y = x e^{-4x}$$

$$\frac{d}{dx} (e^{-4x} \cdot y) = x e^{-4x}$$

$$\int \frac{d}{dx} (e^{-4x} \cdot y) dx = \int x e^{-4x} dx$$

$$e^{-4x} \cdot y = x \int e^{-4x} dx - \left[\frac{d}{dx} (x) \int e^{-4x} dx \right] dx$$

(5)

$$e^{-4x} \cdot y = \frac{x e^{-4x}}{4} + \int \frac{e^{-4x}}{4} dx$$

$$e^{-4x} \cdot y = \frac{-x e^{-4x}}{4} + \frac{1}{4} \int e^{-4x} dx$$

$$= \frac{-x e^{-4x}}{4} - \frac{1}{16} e^{-4x}$$

$$e^{-4x} \cdot y = e^{-4x} \left(\frac{-x}{4} - \frac{1}{16} \right)$$

$$y = e^{4x} \cdot e^{-4x} \left(\frac{-x}{4} - \frac{1}{16} \right)$$

$$\boxed{y = -\left(\frac{x}{4} + \frac{1}{16} \right) + C}$$

Ans