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IDe : 15874

subject linear algebra

Q1

Solve by Gaussian elimination.

ID = 15874

= $x_1 - 8x_2 + x_3 = 0,$

$2x_2 - 8x_3 = 8$

$5x_1 - 5x_3 = 10$

Ans =

$x_1 = 1, x_2 = 0, x_3 = -1$

steps

$$\begin{bmatrix} x_1 - 8x_2 + x_3 = 0 \\ 2x_2 - 8x_3 = 8 \\ 5x_1 - 5x_3 = 10 \end{bmatrix}$$

Write a matrin with the coeffients and solutions

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

Swap matrix Row: $R_1 \leftrightarrow R_3$

$$= \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & 2 & -8 & 8 \\ 1 & -8 & 1 & 0 \end{pmatrix}$$

Cancel leading coeffient in Row R_3 by performing

$$= \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & 2 & -8 & 8 \\ 0 & -8 & 2 & -2 \end{pmatrix}$$

swap matrix row $R_2 \leftrightarrow R_3$

$$= \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & -8 & 2 & -2 \\ 0 & 2 & -8 & 10 \end{pmatrix}$$

Cancel leading coeffient in Row R_3 by performing $R_3 \leftarrow R_3 + \frac{1}{4} \cdot R_2$

$$=0 \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & -8 & 2 & 2 \\ 0 & 0 & -\frac{15}{2} & \frac{15}{2} \end{pmatrix}$$

Date:

$$\begin{pmatrix} 5 & 0 & -5 \\ 0 & -8 & 2 \\ 0 & 0 & \frac{15}{2} \end{pmatrix}$$

multiply matrix row by constant $R_3 \leftarrow \frac{2}{15} \cdot R_3$

$$= \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & -8 & 2 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Cancel leading coefficient in row R_2 by performing $R_2 \leftarrow R_2 - 2 \cdot R_3$

$$= \begin{pmatrix} 5 & 0 & -5 & 10 \\ 0 & -8 & 8 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Date:

Cancel leading coefficient in row R_1 by performing

$R_1 = R_1 - 5 \cdot R_3$

$$= \begin{pmatrix} 5 & 0 & 0 & 5 \\ 0 & -8 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Multiply matrix row by constant

$R_2 \leftarrow -\frac{1}{8} \cdot R_2$

$$= \begin{pmatrix} 5 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Multiply matrix row by constant $R_1 \leftarrow \frac{1}{5} \cdot R_1$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

Q2 inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4^{th}\text{-ID} \\ 5 & -2 & 7 \end{bmatrix}$

ID # 15874

by adjoint method.

Solution As Given

As Given

$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{bmatrix}$, where; 4th-ID # 7

calculate the determent

$\therefore |A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{vmatrix}$, Expand by R_1

$|A| = 3(-7+14) - 4(14-35) + 5(-4+5)$

$|A| = 3(7) - 4(-21) + 5(1)$

$|A| = 21 + 84 + 5$

$|A| = 110$

PTO

Now

calculate the adjoint

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 7 \\ 5 & -2 & 7 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -1\times7-7\times(-2) & 2\times7-7\times5 & 2\times(-2)-(-1)\times5 \\ 4\times7-5\times(-2) & 3\times7-5\times5 & 3\times(-2)-4\times5 \\ 4\times7-5\times(-1) & 3\times7-5\times5 & 3\times(-1)-4\times2 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -7+14 & 14-35 & -4+5 \\ 28+10 & 21-25 & -6-20 \\ 4+5 & 21-10 & -3-8 \end{bmatrix}$$

$$\text{Adj } A = \begin{pmatrix} +(7) & -(-21) & +(1) \\ -(38) & +(-4) & -(-26) \\ +(9) & -(11) & +(-11) \end{pmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 7 & 21 & 1 \\ -38 & -4 & 26 \\ 9 & -11 & -11 \end{bmatrix}$$

PTO

$$\text{Adj } A = \begin{bmatrix} 7 & -38 & 9 \\ 21 & -4 & -11 \\ 1 & 26 & -11 \end{bmatrix}$$

$$\text{Inverse of } A = \frac{1}{|A|} \cdot \text{adj } A$$

$$= \frac{1}{110} \begin{bmatrix} 7 & -38 & 9 \\ 21 & -4 & -11 \\ 1 & 26 & -11 \end{bmatrix} = \text{Result}$$

Q#03:- Linear System by Guass-Jordan Method;

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Sol:-

The above given linear system can be written in matrix form as;

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right]$$

$$\downarrow$$

$$\therefore \begin{bmatrix} 2 & 2 & 4 \\ 1 & 3 & 2 \\ 3 & 2 & -3 \end{bmatrix}$$, solving by Guass Jordan,

I: Echelon form:

P.T.O

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$$R_1 \longleftrightarrow R_3$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 1 & 3 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

$$R_2 - \tfrac{1}{3}R_1 \longrightarrow R_1$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 3 \\ 2 & 2 & 4 \end{bmatrix}$$

$$R_3 - \tfrac{2}{3}R_1 \longrightarrow R_3$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 3 \\ 0 & 2/3 & 6 \end{bmatrix}$$

$$R_3 - \tfrac{2}{7}R_2 \longrightarrow R_3$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 3 \\ 0 & 0 & 36/7 \end{bmatrix}.$$

Now, further Reduce the matrix to Reduced Row Echelon form;

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$$\therefore \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 3 \\ 0 & 0 & 36/7 \end{bmatrix}$$

$$\tfrac{7}{36} \cdot R_3 \longrightarrow R_3$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 - 3R_3 \longrightarrow R_2$$

$$\sim \begin{bmatrix} 3 & 2 & -3 \\ 0 & 7/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 + 3 \cdot R_3 \longrightarrow R_1$$

$$\sim \begin{bmatrix} 3 & 2 & 0 \\ 0 & 7/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tfrac{3}{7} \cdot R_2 \longrightarrow R_2$$

$$\sim \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(P.T.O)

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$R_1 - 2 \cdot R_2 \rightarrow R_1$

$$\sim \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\frac{1}{3} \cdot R_1 \rightarrow R_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

∴ Hence, the linear system solved by Guass-Jordan method Gives us;

$x = 1, \ y = 1, \ z = 1$

= Result

Q#05 To describe the sol set if the Given homogeneous system has non-trivial Solution;

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

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Sol:

As Given;

$$3x_1 + 5x_2 - 4x_3 = 0$$

$$-3x_1 - 25x_2 + 4x_3 = 0$$

$$6x_1 + x_2 - 8x_3 = 0$$

In Matrix form;

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

we will solve the solution of these equations by Gaussian Elimination;

∴ Reduce matrix to Row Echelon form;

i.e: $$\begin{bmatrix} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{bmatrix}$$

$$R_1 \longleftrightarrow R_3$$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ -3 & -25 & 4 & 0 \\ 3 & 5 & -4 & 0 \end{bmatrix}$$

$$R_2 + \tfrac{1}{2} \cdot R_1 \longrightarrow R_2$$

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$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 3 & 5 & -4 & 0 \end{bmatrix}$$

$$R_3 - 1/2 \cdot R_1 \longrightarrow R_3$$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 9/2 & 0 & 0 \end{bmatrix}$$

$$R_3 + 9/49 \cdot R_2 \longrightarrow R_3$$

$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Now,

Reduce Matrix to Reduced Row Echelon Form;

$$\begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & -49/2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$-\frac{2}{49} \cdot R_2 \longrightarrow R_2$$

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$$\sim \begin{bmatrix} 6 & 1 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 1 \cdot R_2 \rightarrow R_1$

$$\sim \begin{bmatrix} 6 & 0 & -8 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{6} \cdot R_1 \rightarrow R_1$

$$\sim \begin{bmatrix} 1 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Zero Row in Reduced Matrix indicates infinite solutions.

i-e:

$x_1 - 4/_3 x_3 = 0 \Rightarrow x_1 = 0$

$x_2 = 0$

$x_3 = 0$

substitute

Also;

Ig we see, the Rank of Matrix is 2 and the number of unknowns in system are 3.

So; $2 < 3$. According to Condition, the system has non-trivial solutions.