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Subject :- LINEAR ALGEBRA

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Q No 1:- Compute Adjoint of

(i) $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$.

Sol:

$\begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix}$.

even = +
odd = −

$A = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$A11 = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6 - 1 = 5$

$A12 = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4 - 3 = 1$

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$$A13 = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 2 - 9 = -7$$

$$A21 = \begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = \cancel{20}\ 4 - 4 = 0$$

$$A22 = \begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix} = 2 - 12 = -10$$

$$A23 = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = -5$$

$$A31 = \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = 2 - 12 = -10$$

$$A32 = \begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} = 1 - 8 = -7$$

$$A33 = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 6 - 4 = 2$$

Cofactors of Matrix A:-

$$\begin{bmatrix} 5 & -1 & -7 \\ 0 & -10 & 5 \\ -10 & 7 & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -10 & 7 \\ -7 & 5 & 2 \end{bmatrix}$$

(ii) $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$B = \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

$$B11 = \begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = -8-16 = -24$$

$$B12 = \begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = 16-40 = 24$$

$$B13 = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = -4-5 = -9$$

$$B21 = \begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = 32-10 = 22$$

$$B22 = \begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = 24-25 = -1$$

$$B23 = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -6-20 = -26$$

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$$B_{31} = \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = 32-(-5) = 37$$

$$B_{32} = \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = 24-10 = 14$$

$$B_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3-8 = -11.$$

Cofactor of B:-

$$\begin{bmatrix} -24 & 24 & -9 \\ 22 & -1 & 26 \\ 37 & 14 & -11 \end{bmatrix}$$

$$B^T = \begin{bmatrix} -24 & 22 & 37 \\ 24 & -1 & 14 \\ -9 & 26 & -11 \end{bmatrix}$$

A

Q No 2:- Find the Cofactors of A_{21}, A_{31}, A_{33} if.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Sol:-

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -(-4-9) = 13$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = (-2-9) = -11$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = 4+4 = 8$$

Cofactors of matrix A:- =

$$\begin{bmatrix} 13 \\ -11 \\ 8 \end{bmatrix}$$ Ans.

Q No 3:-

Find Eigen value &

Eigen vectors.

Sol:-

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \; \& \; I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0.$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

8

$$1\lambda^3 - \left[\begin{smallmatrix}\text{Sum of}\\ \text{diagonal}\\ \text{elements}\end{smallmatrix}\right]\lambda^2 + \left(\begin{smallmatrix}\text{Sum of}\\ \text{diagonal}\\ \text{minors}\end{smallmatrix}\right)\lambda - |A| = 0$$

$$\therefore \lambda^3 - 7\lambda^2 + 14\lambda - 7 = 0$$

$$\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 6-2 = 4$$

So, $\lambda = \boxed{1, 2, 3}$

$\downarrow$

Eigen value.

$$\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 4+1 = 5$$

$$\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6-1 = 5$$

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Eigen vectors :-

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

by Crammer rule :-

$$x_1 - x_2 - x_3 = 0 \quad — (1)$$

$$x_1 \quad x_2 \quad 2x_3 = 0 \quad — (2)$$

$$x_1 = -x_2 = x_3$$

$$\begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} \quad \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} \quad \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$\frac{x_1}{-2-1} = \frac{\cancel{-}x_2}{\cancel{-}2+1} = \frac{x_3}{1+1}$$

$$\frac{x_1}{-3} = \frac{-x_2}{3} = \frac{x_3}{2}$$

(10)

$$X_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}$$

let consider $\lambda = 2$.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & +1 & 2 \\ -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$x_1 - x_2 - x_3 = 0$$

$$x_1 - \cancel{+} x_2 - 2x_3 = 0.$$

Crammer Rule,.

$$x_1 = -x_2 = x_3.$$

$$\begin{vmatrix} -1 & -1 \\ -1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}.$$

$$\frac{x_1}{3} = \frac{x_2}{-3} = \frac{x_3}{-2}.$$