NAME: WASIM

ID : 11758

Subject: Differential equation

Date : 28/09/2020

Q#1

ANs 1 Section A.

A Linear System of equation $Ax = b$ is Called homogeneus if $B = 0$, and non homogeneus if $b \neq 0$ Notice that $x = 0$ is the Solution of the homogeneus equation
The Solutions of an homogeneus System with 1 and 2 free variables are a line and planes Respectively through the Origin.

OR each such non-homogeneus equation has a Corresponding homogeneus equation $y'' + p(t)y' + q(t)y = 0$
Non-homogeneus Second order Linear equation with Constant Coefficient $ay'' + by' + cy = g(t)$.

Example: $y'' - 8y' - 3y = 3t \cdot 2 + 4t - 5$.

Example: $y'' - 2y' - 3y = 5\cos(2t)$

2)
b)
i) $16y'' + 24y' + 9y = 0$

soln:

Determine the characteristic equation by replacing y'' with r^2, y' with r and y with 1 in the differential equation

$$16r^2 + 24r + 9 = 0$$

Determine the root

$$r = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$$

$$\boxed{r = -\frac{3}{4}}$$

General Solution with the characteristic equation has only 1 Solution.

$$y(t) = C_1 e^{rt} + C_2 t e^{rt}$$

with λ the (double) root of the characteristic equation

Since $\boxed{\lambda = \frac{-3}{4}}$ The general solution is then

$$y(t) = C_1 e^{-3t/4} + C_2 t e^{-3t/4}$$

Hence

$$\boxed{y(t) = C_1 e^{-3t/4} + C_2 t e^{-3t/4}}$$

Ans

Q#o 1) B (ii)

ii) $y'' - 4y' - 12y = 3e^{5x}$

Soln

$y'' - 4y' - 12y = 3e^{5x}$

$\therefore y = C_1 e^{6x} + C_2 - \frac{3}{7} e^{5x}$

Steps

$y'' - 4y' - 12y = 3e^{5x}$

$a(x)y'' - (b(x)y' + C(x) + g(x)$

Find y_h by Solving $y'' - 4y' - 12y = 0$

Find y_p that Satisfies

$y'' - 4y' - 12y = 3e^{5x}$ $y = \frac{3}{7} e^{5x}$

The General Solution

$y = y_h + y_p$

$y = y_h + y_p$

$$\boxed{y = C_1 e^{6x} + C_2 e^{-2x} - \frac{3}{7} e^{5x}}$$

Ans.

Q#C2

i)

$$2y'' + 5y' + 3y = 0 \quad y(0) = 3 \quad y'(0) = -4$$

Soln

I have come up with

$$y = C_1 e^{-\frac{3}{2}x} + C_2 e^{-x}$$

$$y' \Rightarrow -\frac{3}{2} C_1 e^{-\frac{3}{2}x} - C_2 e^{-x}$$

$$y(0) = \begin{matrix} C_1 \\ + \\ C_2 \end{matrix} = 3$$

$$y'(0) = \frac{C_1}{C_2} = -4$$

Now Add two equations

$$-\frac{1}{2} C_1 = -1$$

$$\boxed{C_1 = 2}$$

and

$$\boxed{C_2 = 1}$$

Q No2 (ii)
B) $2y'' - 5y' - 3y = 0$

$y(0) = 3$, $y'(0) = 4$

Let $y = e^{Mx}$ be a Solution So that the eq is

$2M^2 - 5m - 3 = 0$

$(2m+1)(m-3) = 0$

$M = 0.5$ and $= 3$

$y = a(e^{-0.5x}) + b(e^{3x})$

$1 = a(e^{0}) + b(e^{0})$

$y' = -0.5a(e^{-0.5x}) + 3b(e^{3x})$

$4 = -0.5a(e^{0}) + 3b(e^{0})$

Solving the System of two equation

$a = -2 \quad b = 1$

$$\boxed{y = -2(e^{-0.5}) + e^{3x}}$$

Ans

Q#3

A

i) $f(t) = 6e^{(-5t)} + e^{(3t)} + 5^{(t^3)} - 9$

Soln:

$$f(t) = 6e^{-5t} + e^{3t} + 5^{t^3} - 9$$

$$F(s) = \mathcal{L}\{f(t)\} = 6\mathcal{L}\{e^{-5t}\} + \mathcal{L}\{e^{3(t)}\}$$

$$+ 5\mathcal{L}\{t^3\} - 9\mathcal{L}\{1\}$$

$$= 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

Hence

$$\boxed{\frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}}$$

Ans.

ii)

Sol

G.

s.

QNo (8)

Define Laplace Transformation along with example

Laplace Transform is an integral transform that convert a function of a real variable t (often time) to a function of a complex variable s the Transform has many application in science and Engineering because it is a Tool for Solving Differential equation In partial it perform a Transform Differential equation and convolution into Multiplication

for example

Let $f(t) = 1$ when $t \geq 0$ find $F(s)$

Solution

From (1) we obtain by integration

Q#3)

ii) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Soln

$$G(s) = 4\,\frac{s}{s^2+(4)^2} - 9\,\frac{4}{s^2+(4)^2} + 2\,\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

Hence

$$\boxed{\frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}}$$

Ans

iii) $h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

Soln

$$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{(s-3)^2+(6)^2}$$

$$= \boxed{\frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}}$$

Ans

Q# 4) ii)

ii) $y'' - 4y' = e^{3t}$

soln:

$$s^2 y(s) - sy(0) - y'(0) - 4sy(s) + 4y(0) = \frac{1}{s-3}$$

$$s^2 y(s) - s \times 0 - 0 - 4sy(s) + 4 \times 0 \Rightarrow \frac{1}{s-3}$$

$$s^2 y(s) - 4sy(s) = \frac{1}{s-3}$$

$$y(s)(s^2 - 4s) = \frac{1}{s-3}$$

$$y(s) = \frac{1}{s(s-4)(s-3)}$$

Step #02

$$\frac{1}{s(s-4)(s-3)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s-3}$$

$$1 = A(s-4)(s-3) + B(s)(s-3) + C(s)(s-4)$$

Take $s=0$

$1 = A(-4)(-3)$

$$\boxed{A = \frac{1}{12}}$$

Take $s=4$

$1 = B \times 4 \times 1$

Take $s=3$

$1 = C \times 3 \times -1$

$$\boxed{C = -\frac{1}{3}}$$

Hence

$$Y(s) = \frac{1}{12}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4} - \frac{1}{3}\frac{1}{s-3}$$

Hence

$$\boxed{Y(s) = \frac{1}{12}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4} - \frac{1}{3}\frac{1}{s-3}}$$

Ans

Q#4 iii) Step

ii) $y'' + 3y' + 2y = e^{-t}$ $y(0) = 0$ $y'(0) = 0$

Sol:

$y'' + 3y' + 2y$ $y(0) = 0$, $y'(0) = 0$

Y

Take the Laplace Transformation

$$L[y''] + 3L[y'] + 2L[y] = L[e^{-t}]$$

$$s^2 Y(s) - sY(0) - y'(0) + 3[sY(s) - y(0)] + 2Y(s) = \frac{1}{s+1}$$

Step #02.

put initial value and Simplify

$$s^2 Y(s) + 3[sY(s)] + 2Y(s) = \frac{1}{s+1}$$

$$Y(s)[s^2 + 3s + 2] = \frac{1}{s+1}$$

$$Y(s) = \frac{1}{s+1[s^2 + 3s + 2]}$$

$$= \frac{1}{(s+1)^2 (s+2)}$$

13

Step # 03.

$$Y(s) = -\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$$

$$\boxed{y(t) = -e^{-t} + te^{-t} + e^{-2t}}$$ Ans