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| ID | 7655 |
| Name | Syed USAMA |
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| Submitted to | Shumail Mazhar |
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Q1) $f(t) = 1 + t \quad -\pi < t < \pi$

here we use the formula

Sol)

$\rightarrow \quad f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad \leftarrow eq(1)$

$\rightarrow \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$

$\rightarrow \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$

$\rightarrow \quad a_0 = \frac{1}{2\pi} \left| t + \frac{t}{2} \right|_{-\pi}^{\pi}$

$\rightarrow \quad a_0 = \frac{1}{2\pi} \left( \pi - (-\pi) + \frac{\pi^2}{2} - \left( -\frac{\pi^2}{2} \right) \right)$

$\rightarrow \quad a_0 = \frac{1}{2\pi} \left( 2\pi + \frac{2\,\ ^2}{2} \right)$

$\rightarrow \quad \boxed{a_0 = \frac{1}{2\pi} \left( 2\pi + \pi^2 \right)}$

$\rightarrow \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$

$a_n = \frac{1}{\pi} \left( (1+t) \frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left( \frac{\sin nt}{n} \frac{d}{dt}(1+t) \right. \right.$

$\Rightarrow a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \left(\frac{\sin nt \cdot d}{n\ dt}(1+t)\right)\right.$

$\Rightarrow a_n = \frac{1}{\pi}\left((1+t)\frac{\sin \cancel{n}t}{\cancel{n}} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$

$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$

$\Rightarrow \boxed{a_n = 0}$

$b_n = \cancel{\frac{1}{\pi}}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt)$

$b_n = \cancel{\frac{1}{\pi}}\left(\frac{(1+t)(-\cos nt)}{n}\Big|_{-\pi}^{\pi} + \frac{\sin nx}{n^2}\Big|_{-\pi}^{\pi}\right)$

$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+\pi)(\cos n\pi)\right)$

$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi\right) - \left(\cos n\pi + \pi\cos n\pi\right)$

$b_n = \frac{-1}{n\pi}\left(2\pi\cos n\pi\right)$

$= \cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So eq becomes

$$F(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \operatorname{Sin} t$$

Q2)

Calculate ......

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Solve) Step 1;

$\therefore A =$ Given matrix

We have;

$$(A - \lambda I)x = 0$$

Step 2;

We have the characteristic equation given by

$$(A - \lambda I) = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$7 \quad \begin{bmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{bmatrix} = 0$$

Step # 3

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{diagnol elem}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{diagnol} \\ \text{minors}\end{matrix}\right| \lambda - |A| = 0$$

(B)

sum of diagnol elements $= 1+1+2 = 4$

Sum of diagnol minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= -6 + 2 + 1 = -3$

by putting in eq(B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{———} (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$

$$= -6 + 6 = 0$$

by putting values in (6) eq

$$(\lambda^3 - 4\lambda^2 - 3\lambda - 0) = 0$$

$$(\lambda^3 - 4\lambda^2 - 3) = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Using Quadratic formula

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

We have

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

Q3)

$$5x + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 0$$

Sol)

$$\left(\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & 9 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right) \begin{array}{l} R_2 - 5R_1 \\ R_3 - 4R_1 \\ R_4 - R_1 \end{array}$$

$$= \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 5 & -6 & -4 & -3 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right) \; 1/5 \, R_2$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & -1 \\ 0 & 0 & -\frac{23}{5} & 6/5 & -1/5 \end{array}\right) \begin{array}{l} R_3 - 5R_2 \\ R_4 - 2R_2 \end{array}$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 6/5 & -1/5 \end{array}\right) \left(\frac{1}{-15}\right) R_3$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 113/75 & 8/75 \end{array}\right) R_4 + \frac{23}{5} R_3$$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 9/5 & -3/5 & -2/5 \\ 0 & 0 & 1 & 1/15 & 1/15 \\ 0 & 0 & 0 & 1 & 8/113 \end{array}\right) \left(75/113\right) R_4$$

$$x - y + 2z + m = 1 \quad (1)$$

$$y + 9/5 z - 3/5 m = -2/5 \quad (2)$$

$$z + 1/15 m = 1/15 \quad (3)$$

$$m = 8/113 \quad (4)$$

Putting values of m in eq 1

$$z + \frac{1}{15}\left(\frac{8}{113}\right) = \frac{1}{15}$$

$$z + \frac{8}{1695} = \frac{1}{15}$$

$$z = \frac{113 - 8}{1695} = \frac{105}{1695}$$

Putting values in eq(3)

$$y + \frac{9}{5}\left(\frac{105}{1695}\right) - \frac{3}{5}\left(\frac{8}{113}\right) = \frac{-2}{5}$$

$$y + \frac{945}{8475} - \frac{24}{565} = \frac{-2}{5}$$

$$y = \frac{-2}{5} - \frac{585}{8475}$$

$$y = \frac{-3390 - 585}{8475}$$

$$y = -\frac{3975}{8475}$$

Putting values in eq (1)

$$x + \frac{3975}{8475} + \frac{210}{1695} + \frac{8}{113} = 1$$

$$x + \frac{3975 + 1050 + 600}{8475} = 1$$

$$x + \frac{5625}{8475} = 1$$

$$x = 1 - \frac{5625}{8475}$$

$$x = \frac{2850}{8475} = \frac{570}{1695}$$

$$x = \frac{114}{339}$$

Q4)

Verify
$$u(x,t) = \sin(x+2t)$$
is a solution of the one dimension wave equation.

Sol)
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x^2}$$

$$u(x,t) = \sin(x+\gamma t) \quad \text{is}$$

Solution of $\frac{\partial^2 u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x}$

it will satisfy the above eq

$$\frac{\partial u}{\partial t} = \cos(x+\partial t)\frac{d}{dt}(x+\gamma t)$$

$$\frac{\partial u}{\partial t} = \partial \cos(x+\gamma t)$$

again $\frac{\partial^2 u}{\partial t^2} = -\partial \sin(x+\gamma t)\frac{\partial t}{\partial t}(x+\gamma t)$

$\rightarrow \frac{\partial^2 u}{\partial t^2} = -4 \sin(x + \gamma t) — A$

Now

$$\frac{\partial u}{\partial x} = \cos(x + \gamma t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x + \gamma t)$$

$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\sin(x + \gamma t) — (B)$

Comp A and B

$\therefore c = 2$

$\rightarrow -4 \sin(x + \gamma t) = -c^2 \sin(x + \gamma t)$

$\rightarrow -4 \sin(x + \gamma t) + c^2 \sin(x + \gamma t) = 0$

This is possible ie $c = \pm 2$

$\rightarrow -4 \sin(x + \gamma t) + (\pm 2)^2 \sin(x + \gamma t) = 0$

$\boxed{0 = 0}$ This

$$u(x, t) = \sin(x + \gamma t)$$

(3)

s we know that one-dimensional
wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

For the orbitrary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitrary Constant

$c = 2.$