Course Title : Electrical Network Analysis

Module : 4^{th}

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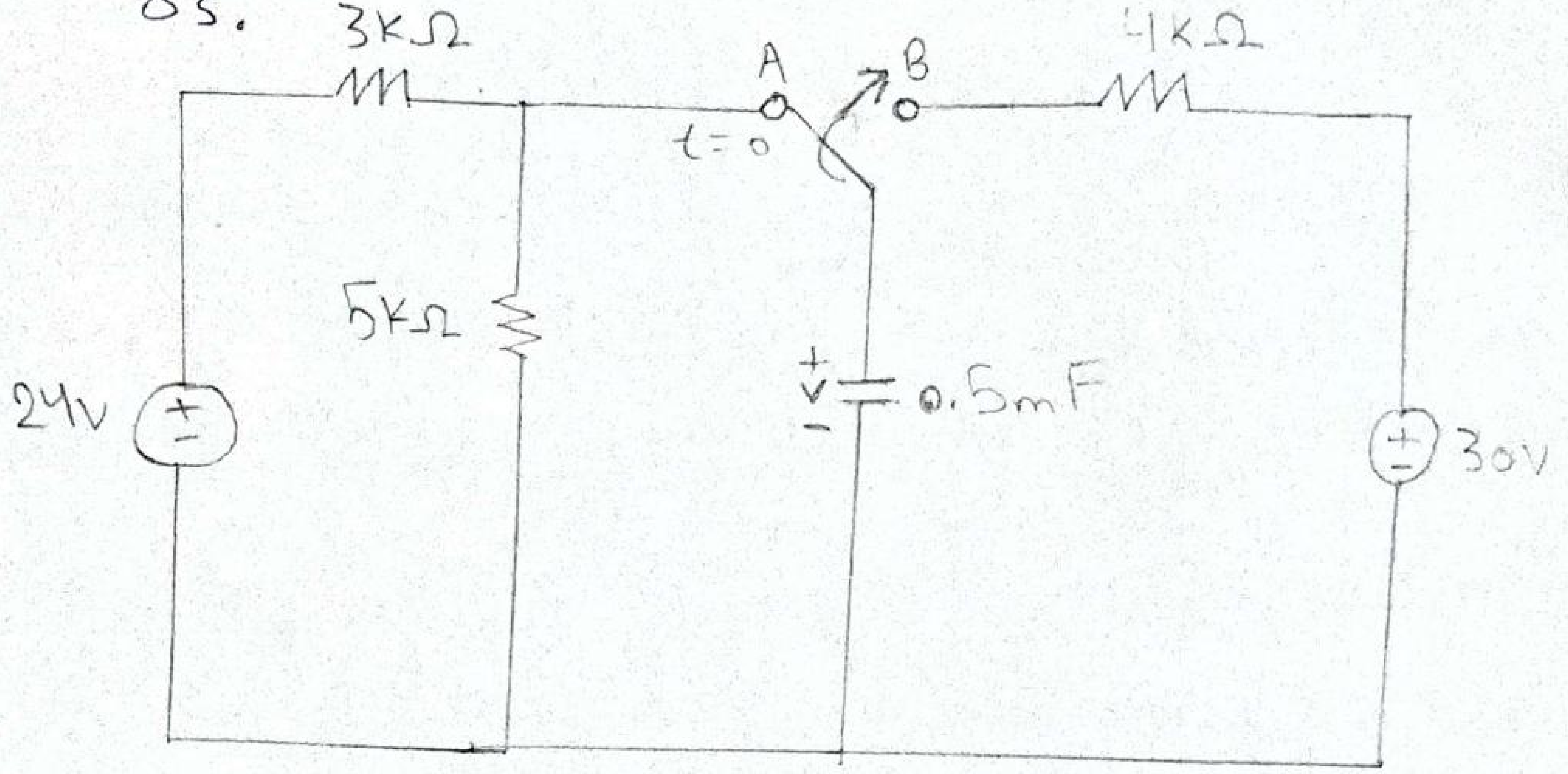
Q1 The switch in Fig. 1 has been in Position A for a long time. At $t=0$ the switch moves to B. Determine $v(t)$ for $t>0$ and calculate its value at $t=2s$ and $8s$.



Sol:

For $t<0$, the switch is at Position A. The capacitor acts like an open circuit to dc, but v is same as the voltage across the 5KΩ resistor. Hence the voltage across the capacitor just before $t=0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15V$$

Using the fact that the capacitor voltage can not change instantaneously

$$v(0) = v(0^-) = v(0^+) = 15V$$

For $t > 0$, the switch is in Position B

The Thevenin resistance Connected to the Capacitor is $R_{Th} = 4k\Omega$, and the time Constant is

$$\tau = R_{Th} C = 4 \times 10^{3} \times 0.5 \times 10^{-3} = 2s$$

Since the Capacitor acts like an open circuit to dc at steady state, $V(\infty) = 30V$. Thus,

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

$$= 30 + (15 - 30 e^{-t/2}), = (30 - 15 e^{-0.5t}) V$$

At $t = 2s$

$$V(2) = (30 - 15 e^{-0.5(2)}) V$$

$$V(2) = (30 - 15 e^{-1}) V$$

$$V(2) = (30 - 5.517) V$$

$$V(2) = 24.483 V$$

$$V(2) = 24.48 V$$

At $t = 8s$

$$V(8) = (30 - 15 e^{-0.5(8)}) V$$

$$V(8) = (30 - 15 e^{-4}) V$$

$$V(8) = 30 - 15(0.0183)$$

$$V(8) = 30 - 0.2745$$

$$V(8) = 29.7255$$

$$V(8) = 29.72 V$$

Q2

Determine the inductor current for both $t > 0$ and $t < 0$ for the circuit in Fig 2

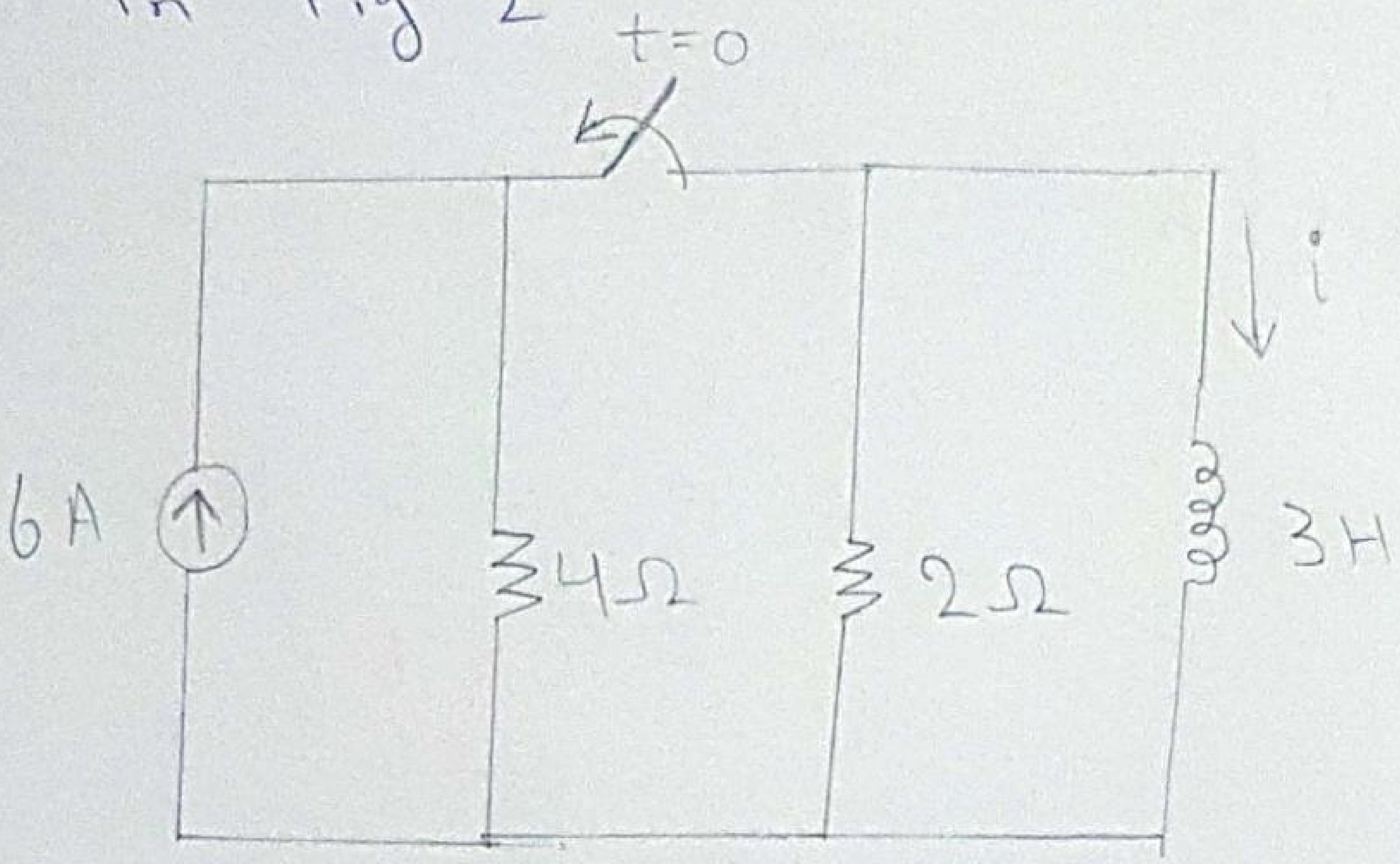


Fig 2

Sol:

For $t < 0$

The switch is closed and inductor acts as short circuit therefore inductor current is

$$i = 6\,A$$

For $t > 0$

The switch is opened and time constant

$$\tau = L/R$$

$$\tau = \frac{3}{2}$$

Now the inductor current

$$i(t) = 6e^{-t/\tau}$$

$$i(t) = 6e^{-t/3/2}$$

$$i(t) = 6e^{-\frac{2t}{3}}u(t) \text{ A}$$

Q3 A Series RLC circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Find the response when $L = 0.5H$, $R = 4\Omega$ and $C = 0.2F$. Let $i(0) = 1$, $di(0)/dt = 0$

Sol

Given $L = .5H$

$R = 4\Omega$

$C = 0.2F$

$i(0) = 1A$

$$\frac{di(0)}{dt} = 0$$

The step response of the branch voltage of the given RLC circuit is described by:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divide by L,

$$\frac{d^2i}{dt^2} + \frac{R\,di}{L\,dt} + \frac{i}{LC} = \frac{10}{L}$$

For right hand side of the equation Multiply by $\frac{C}{C}$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10}{LC}$$

And $C = 0.2F$, thus

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute,

$$\frac{d^2 i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \quad \ldots (1)$$

The general equation for a source free series RLC circuit is given by:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{I_s}{LC} \quad \ldots (2)$$

Compare both the equations 1 & 2

$$\frac{R}{L} = 8 \quad \rightarrow (3)$$

$$\frac{1}{LC} = 10 \quad \ldots (4)$$

$$\frac{I_s}{LC} = 20 \quad \ldots (5)$$

From (3), α is given by

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4\,\text{rad/s} \quad \ldots (6)$$

The natural frequency ω_0 is given by

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From (4)

$$\omega_0 = \sqrt{10}\ \text{rad/s}$$

From (6) and (7)

From ω_0 and α

$$\because \alpha > \omega_0$$

$\therefore$ The circuit is over damped

The roots of the characteristic equation is given by,

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$
$$= -4 + \sqrt{6}\ \text{rad/s}$$

And

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 - \sqrt{4^2 - \sqrt{10}^2}$$
$$= -4 - \sqrt{6}\ \text{rad/s}$$

From $\frac{I_s}{LC} = 20$ The steady state

current is given by

$$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A \quad \rightarrow (8)$$

The current for the over damped case is given by :

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \quad \rightarrow (9)$$

Substitute $t = 0$

$$i(0) = I_s + A_1 + A_2$$

Substitute, $1 = 2 + A_1 + A_2$

Thus, $A_1 + A_2 = -1 \quad \rightarrow (10)$

From (9) $i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 S_1 e^{s_1 t} + A_2 S_2 e^{s_2 t}$$

Substitute $t = 0$

$$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$$

Substitute the values,

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \quad -(11)$$

Solve for $A_1 + A_2 = -1$ and $(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0$
means solving (10) & (11)

$$A_1 = -1.316$$

$$A_2 = 0.316$$

Substitute $i(t)$ (9)

$$i(t) = 2 - 1.316e^{(-4+\sqrt{6})t} + 0.316e^{(-4-\sqrt{6})t} \text{ A}$$

From (9) damped current $i(t)$ find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Substitute $t = 0$

$$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$$

Substitute the values,

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0$$

Solve $(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0$ and
$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0$

$$A_1 = -1.316$$
$$A_2 = 0.316$$

Substitute in $i(t)$

$$i(t) = 2 - 1.316\,e^{(-4+\sqrt{6})t} + 0.316\,e^{(-4-\sqrt{6})t} \text{ A}$$

Q4: A Series RLC circuit has $R = 100\Omega$

$L = 240H$ and $C = 10mF$. if the input voltage is $v(t) = 10\cos 2t$, Find the current flowing through the circuit.

Sol: Input voltage is $v(t) = 10\cos 2t\ V$

Hence

$$\text{Amplitude} = V_m = 10V$$

$$\text{Angular frequency } \omega = 2 \text{ rad/s}$$

$$\text{Phase angle}, \phi = 0^\circ$$

So Phasor for the voltage $v(t)$

$$V(t) = 10\angle 0^\circ V$$

Now for Inductive reactance

$$X_L = \omega L$$

So $\omega = 2 \text{ rad/s}$, $L = 240H$

$$X_L = (2)(240)$$

$$X_L = 480\Omega$$

Now For capacitive reactance

$$X_C = \frac{1}{\omega C}$$

$$\omega = 2 \text{ rad/s}, \quad C = 10mF$$

$$= \frac{1}{2(10\times10^{-3})}$$

$$= \frac{1}{2\times10^{-2}}$$

$$= \frac{1\times10^{2}}{2}$$

$$= \frac{100}{2}$$

$$X_C = 50\Omega$$

Now for Impedence

$$Z = R + jX_L - jX_C$$

$R = 100\Omega$, $X_L = 480\Omega$, $X_C = 50\Omega$

Putting in equation

$$Z = (100 + 480j - 50j)\Omega$$

Represent "Z" in Phasor form

$$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$$

$$Z = \sqrt{10000 + 184900} \angle \tan^{-1}(4.3)$$

$$Z = \sqrt{194900} \angle \tan^{-1}(4.3)$$

$$Z = 441.47 \angle 76.9^{\circ}\Omega$$

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Now for current flowing in the circuit ;

$$i = \frac{v(t)}{Z}$$

$v(t) = 10\angle 0^\circ$, $Z = 441.47\angle 76.90^\circ \Omega$

Putting in equation:

$$i = \frac{10\angle 0^\circ V}{441.47\angle 76.90^\circ \Omega}$$

$$i = \frac{10}{441.47} \angle [0 - 76.90^\circ] A$$

$$= 22.6 \times 10^{-3} \angle -76.90^\circ A$$

$$= 22.6 \angle -76.90^\circ mA$$

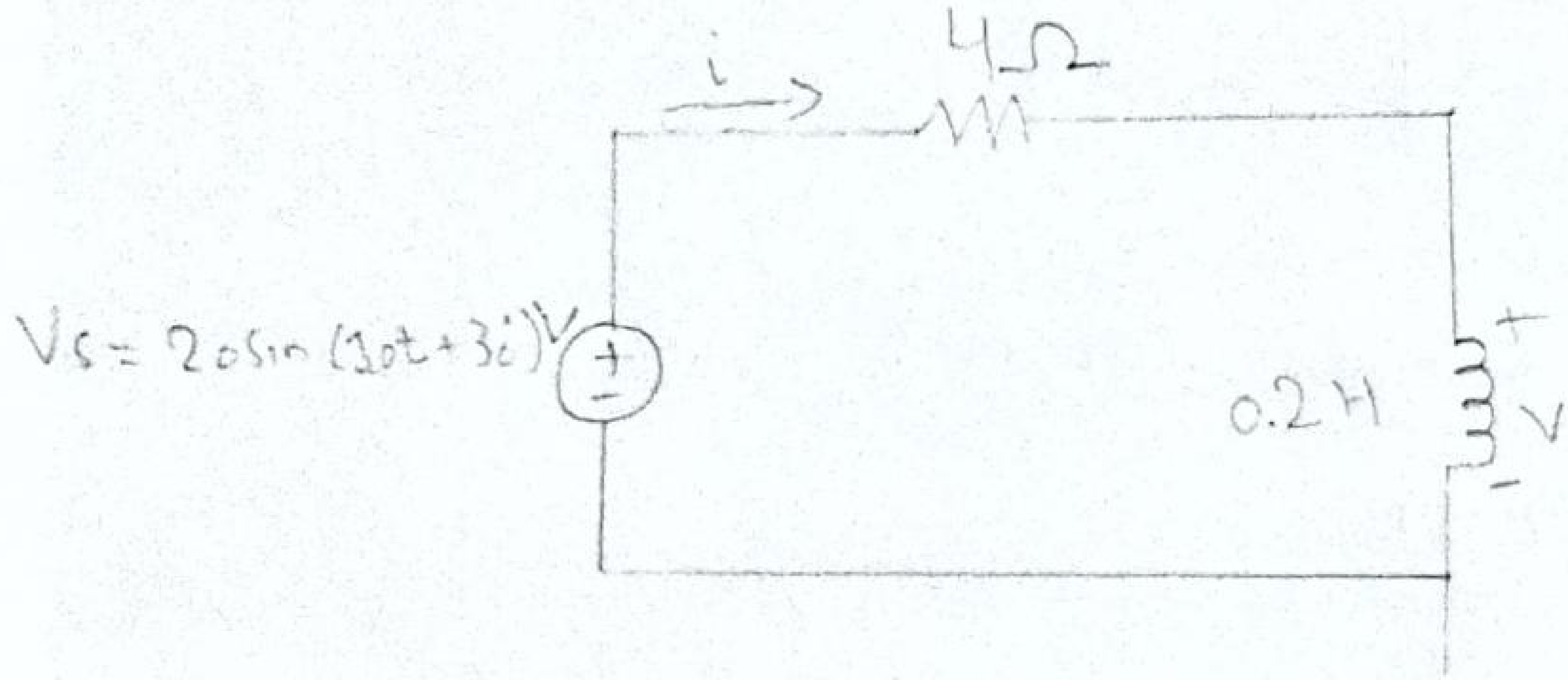
So,

The general expression for 'i'

$$i = 22.6 \cos(2t - 76.90^\circ) mA$$

Question 5

Find $v(t)$ & $i(t)$ in the circuit in the figure



Sol

For $i(t)$

from the voltage source

$$V_s = 20\cos(10t + 30^\circ - 90^\circ)\,V$$

$$V_s = 20\cos(10t - 60^\circ)\,V$$

$$V_s = 20\angle -60^\circ\,V$$

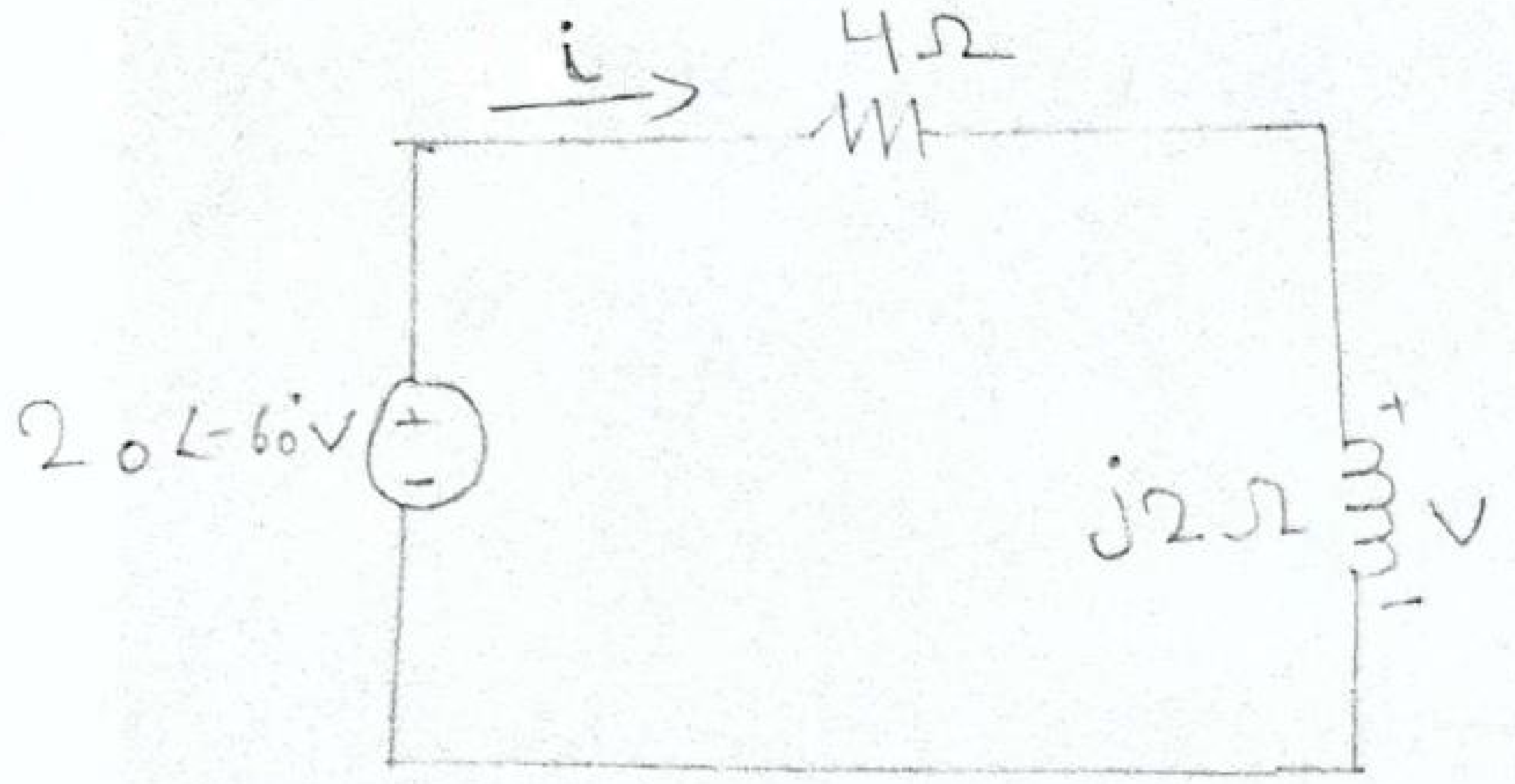
$$\omega = 10 \text{ rad/sec}$$

$$X_L = j\omega L$$

$$0.2H = j \times 10 \times 0.2$$

$$0.2H = j2\,\Omega$$

Given circuit can be represented As ;



From the above circuit

$$Z = 4 + j2\,\Omega$$

Hence the above circuit

$$Z = 4 + j2\,\Omega$$

Hence the current is

$$i = \frac{20\angle -60^\circ}{4 + j2}$$

$$i = \frac{20\angle -60^\circ}{\sqrt{4^2+2^2}\angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$i = \frac{20\angle -60^\circ}{4.472\angle 26.57^\circ}$$

$$i = 4.472\angle -86.57^\circ$$

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Converting this into time domain

$i(t) = 4.472 \cos(10t - 86.57^\circ)$

$i(t) = 4.472 \sin(10t - 86.57^\circ + 90^\circ)$

$i(t) = 4.472 \sin(10t + 3.43^\circ) A$

For $v(t)$:

From the circuit voltage across the inductor .

$V = j2 \times i$

$V = j2 \times (4.472 < 86.57^\circ)$

Converting Polar form to rectangular form we get ;

$V = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j0.53512$

converting rectangular form to Polar form

$$V = (\sqrt{(8.926)^2 + (0.53512)^2} \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$$

$$V = 8.944 \angle 3.4 V$$

Converting this into the time domain

$$V(t) = 8.944 \cos(10t + 3.4^\circ)$$

$$V(t) = 8.944 \sin(10t + 3.4^\circ + 90^\circ)$$

$$V(t) = 8.944 \sin(10t + 93.4^\circ) V$$