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Section = A

Subject = Calculus

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QNo 1. The function (t) is defined by

$g(t) = 0 \quad t < 0$

$t^2 = 0 \leq t \leq 3$

$2t + 3 \quad 3 < t \leq 4$

$12 \quad t > 4$

a) State any point of discontinuity

b) Find if they exist.

i) $\lim_{t \to 3} g$

Sol

a) To check possibility of the discontinuity function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

Apply limits:

$= \cancel{1} + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t+3$

$= \lim_{h \to 0} 2(1-h)+3$

$= \lim_{h \to 0} 2-2h+3$

Apply limits.

$= 2-2(0)+3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

$\Rightarrow$ Now at $t = 4$

$g(4) = 2(4)+3$

$= 8+3 = 11$

For R.H.L

$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h)+3$

$= \lim_{h \to 0} 2+2h+3$

Apply limits

$= 2+2(0)+3 = 5$

For L.H.L

$\lim_{h \to 0} g(1-h) = 12$

$g(4) =$ R.H.L $\neq$ L.H.L

point of discontinuity is at $t=4$

b) Find, if they exist

1) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L

$\lim_{h \to 3} g(1+h) = \lim (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limit.

$= 1 + 3^2 + 2(3) \Rightarrow 16$

(4)

L.H.L

$$\lim_{h\to 3} g(1-h) = \lim_{h\to 3} 2t+3$$

$$= \lim_{h\to 3} 2(1-h)+3$$

$$= \lim_{h\to 3} 2 - 2h + 3$$

Apply limit

$$= 2-2(3)+3$$

$$= 2-6+3 = -1$$

R.H.L $\neq$ L.H.L (do not exist

since L.H.L is -ve)

QNo2 $y(x) = x^2 + \sin x$

By Macalavrin's Series

$y(x) = x^2 + \sin x$

By Using Macalavrines Formula.

$$f(x) = f(0) + x\, f'(0) + \frac{x^2}{2!} + f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$$

Finding Derivation

$$y(x) = x^2 + \sin x$$

(1) $\frac{d}{dx}(y(x)) = \frac{d}{dx}(x^2 + \sin x)$

$= \frac{dy}{dx} = \boxed{2x + \cos x} \longrightarrow f'$

Now again

(2) $\frac{dy^2}{dy^2} = 2 + (-\sin x)$

$y'' = \boxed{2 - \sin x} \rightarrow f''$

Again

(3) $y''' = 0 - \cos x = \boxed{-\cos x} \rightarrow f'''$

Again

(4) $y^{iv} = -(-\sin x) = \boxed{\sin x} \rightarrow f^{iv}$

Now Putting values in the Function

1. $f(0) = (0)^2 + \sin(0) = 0$

2. $f'(0) = 2x + \cos x$

$= 2(0) + \cos(0) = 0 + 1 = 1$

3. $f''(0) = 2 - \sin x$

$= 2 - \sin(0) = 2 - 0 = 2$

4. $f'''(0) = -\cos x = -\cos(0) = -1$

5. $f^{iv}(0) = \sin(0) = 0$

Now Putting all the values in Maclaurin's Formula.

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \cdots$

$x^2 + \sin x = 0 + x(1) + \frac{x^2}{2!}(2) + \frac{x^3}{3!}(-1) + \frac{x^4}{4!}(0) + \cdots$

Q No 3 = A)

Find y'' given

1 $$1 + xy = x^2 + y^2 \longrightarrow (1)$$

Sol

2 Diff eq (1) b/s w.r.t "x"

$$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$$

3 $$\frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

4 $$0 + \left(x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x\right) = 2x + 2y \cdot \frac{dy}{dx}$$

5 $$x \cdot \frac{dy}{dx} + y(1) = 2x + 2y \cdot \frac{dy}{dx}$$

$$x \cdot y' + y = 2x + 2y \cdot y' \longrightarrow (2)$$

Now Again Differentiating eq (2) b/s w.r.t. "x"

$$\frac{d}{dx}(x \cdot y' + y) = \frac{d}{dx}(2x + 2y \cdot y')$$

$$\frac{d}{dx}(x \cdot y') + \frac{d}{dx}y = \frac{d}{dx} \cdot 2x + \frac{d}{dx}(2y \cdot y')$$

$$\left(x \cdot \frac{d}{dx}y' + y' \cdot \frac{d}{dx}x\right) + \frac{dy}{dx} = 2 + \left(2y \cdot \frac{d}{dx}y' + y' \cdot \frac{d}{dx}2y\right)$$

$$x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (1) + \frac{dy}{dx} = 2 + \left(2y \cdot \frac{d^2y}{dx^2} + \right.$$

$$\left. + \frac{dy}{dx} \cdot 2\frac{dy}{dx}\right)$$

$$x \cdot y'' + y' + y' = 2 + 2y \cdot y'' + y' \cdot 2y'$$

$$x \cdot y'' + 2y' = 2 + 2y''' + 2y''.$$

QNo3 (B)

$$y = x^3 (1+x)^9 e^{6x}$$

Sol

Using lagraithmic Differentation

$$\ln(y) = \ln(x^3 \cdot (i+x)^9 \cdot e^{6x})$$

$$= \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

$$= 3\ln(x) + 9\ln(1+x) + 6x\ln e$$

Now Differentiating each ~~lne~~ term.

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(3\ln(x)) + \frac{d}{dx}(9\ln(1+x) + \frac{d}{dx}(6x.\ln e)$$

$$\frac{1}{y}\frac{dy}{dx} = 3\cdot\frac{1}{x} + 9\cdot\frac{1}{1+x} + \frac{6x}{e} + 6\ln(e)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+2} + \frac{6x}{e} + 6\ln(e)$$

$$\frac{dy}{dx} = 3y(\frac{1}{x} + \frac{3}{1+x} + \frac{2x}{e} + 3\ln(e)$$

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