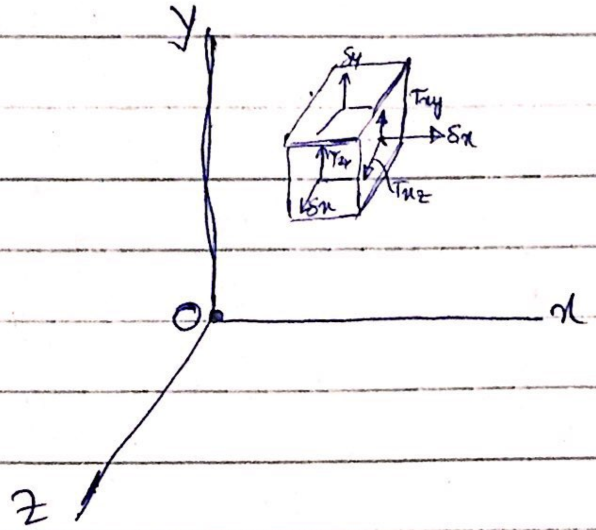
1. Application of Mohr's circle
2. to three dimensional analysis of Stress.

Consider the general 3D state of stress at a point and the transformation of stress from element rotation.



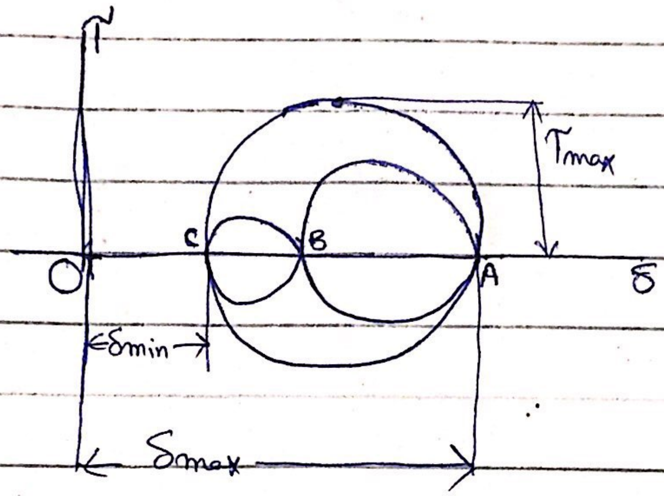
General state of stress at a point may be represented by 6 Components.

σ_x, σ_y, σ_z normal Stresses

τ_{xy}, τ_{yz}, τ_{zx} Shearing Stresses

(Note: $\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$, $\tau_{zx} = \tau_{xz}$).

Transformation of stress for an element rotated around a principal axis may be represented by Mohr's circle.



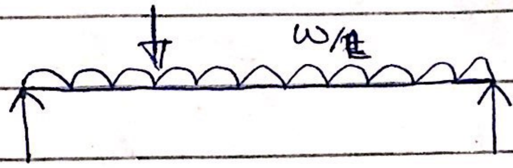
- Points A, B and C represent the principal stresses on the principal planes (Shearing stress is zero).
- The three circles represent the normal and shearing stresses for rotation around each principal axis.
- Radius of the largest circle yields maximum shearing stress

$$\tau_{max} = \frac{1}{2} \left| \sigma_{max} - \sigma_{min} \right|.$$

3. Simple Bending:

Bending will be called as Simple bending when it occurs because of beam Self-load and external load.

This Type of bending results both Shear stresses and normal stresses in the beam.

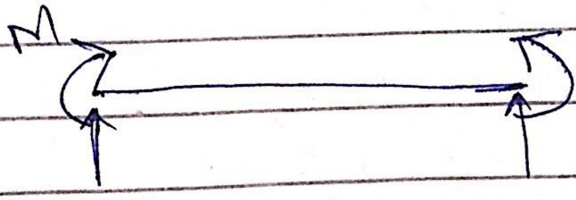


Simple bending.

Pure Bending:

Bending will be called as pure bending when it occurs solely because of Coupling on its end.

In that case there is no Chance of Shear stress in the beam.



Pure bending.

4.

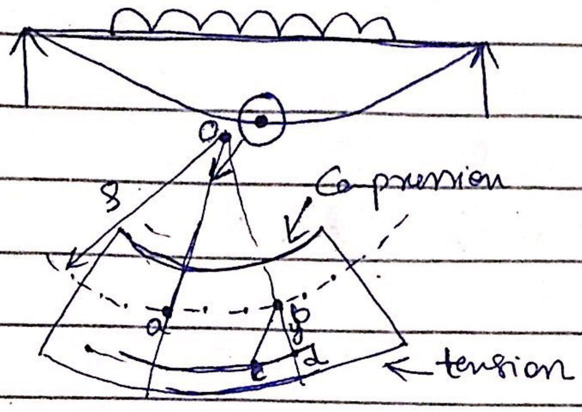
Pure bending occurs only under a constant bending moment (M), since the shear force (V), which is equal to $\frac{dM}{dx} = V$, has to be equal to zero.

In reality, a state of pure bending does not practically exist, because such a state needs an absolutely weightless member. The state of pure bending is an approximation made to derive formulas.

1. In pure bending the axial lines bend to form circumferential lines and transverse lines remain straight and become radial lines.
2. Axial lines that do not extend or contract form a neutral surface.

5. Classic Flexure Equation:

Consider a beam to be loaded as shown



Consider a fiber at a distance 'y' from the neutral axis, because of the beam's curvature as effect of bending moment, the fiber stretched by an amount of 'cd'. Since curvature of the beam is very small, 'bcd' and and 'oba' are considered as similar triangles. The strain on this fiber is

$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

$\rho \rightarrow$ Radius of curvature.

By Hook's law,

$$\varepsilon = \frac{\delta}{E} \text{ or } \delta = E\varepsilon$$

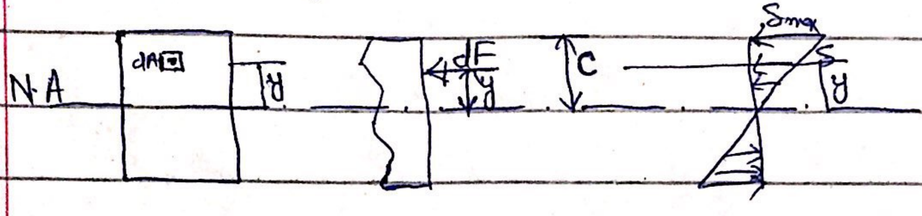
then

$$\frac{\delta}{E} = \frac{y}{\rho}$$

or

$$\sigma = \frac{y}{\rho} E \quad \text{--- (A)}$$

which means that the stress is proportional to the distance 'y' from N.A.



Force acting over 'dA' is

$$dF = \sigma_b \cdot dA = \frac{y}{\rho} E \cdot dA = \frac{E}{\rho} y \cdot dA$$

The resultant of all the elemental moment about N.A must be equal to the bending moment on the section. i.e

$$M = \int dM = \int y \cdot dF = \int \left(\frac{E}{\rho} y \cdot dA\right)(y)$$

$$M = \frac{E}{\rho} \int y^2 dA$$

but $\int y^2 dA = I$ = ~~moment of Inertia~~
= 2nd moment of area

then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M} \quad \text{--- (B)}$$

From eqn (A) $\Rightarrow$ $\rho = \frac{Ey}{\sigma}$

Substituting in eqn (B)

$$\Rightarrow M = \frac{EI}{Ey/\delta}$$

$$\Rightarrow M = \frac{\delta I}{Y}$$

$$\Rightarrow \delta = \frac{MY}{I}$$

and

$$\delta_{max} = \frac{Mc}{I}$$

where

M = Bending Moment

y = distance from N. A

C = Max distance from N. A.

6. Section Modulus:

Section Modulus is a geometric property for a given cross section used in the design of beams or

Flexure members.

Elastic Section Modulus is defined as

$$S = I/y$$

where

I = Second moment of area (area moment of inertia).

y = distance from the N.A to any given fibre.

On Flexure formula,

$$\sigma_{max} = \frac{MC}{I}$$

$$\Rightarrow \sigma_{max} = \frac{M}{I/C}$$

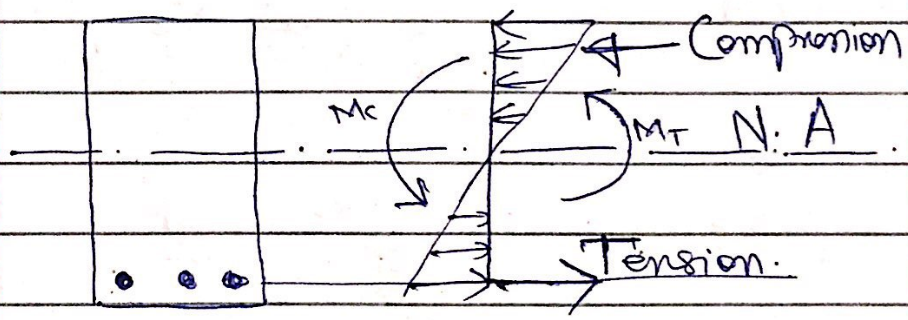
$$\sigma_{max} = \frac{M}{S}$$

where $S = I/C$ = Section Modulus.

7. Application of Bending Equation discussed in No 5.

8. Moment of Resistance:

When a beam bends under load, the horizontal fibres will change in length. The top fibres will become shorter (Compression) and the bottom fibres will become longer (tension).



The tensile and compressive stresses result in a turning effect about the N.A. These are called moment M_t and M_c respectively. In technical terms it is referred as the internal moment of Resistance.

9. Design of Column under Centric Load.

In design of column for Centric loading, cross sectional area is such selected that stresses produced does not exceed allawable stress limit and deformation (lateral buckling) falls within specifications.

$$\text{Stress} = \delta = \frac{P}{A} \leq \delta_{allawable}$$

and

$$\text{deformation} = \Delta = \frac{PL}{AE} \leq \text{Spec.}$$

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10.

Castigliano's First theorem:
The first partial derivative of the total internal energy (Strain Energy) in a structure with respect to any particular deflection component at a point is equal to the force applied at that point and in the direction corresponding to that deflection component.
The first theorem is applicable to linearly or nonlinearly elastic structures in which temperature is constant and supports are unyielding.

2nd theorem:
The first partial derivative of the total internal energy in a structure with respect to the force applied at any point is equal to the deflection at the point of application of that force in the

direction of its line of action.
The second theorem of Castigliano is applicable to linearly elastic structures with constant temperature and unyielding supports.