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Subject: Differential system and equation

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QNO1:-

(a) Estimat the general solution of

$$y' = (x+2)y^2$$

Solution:-

$$y' = (x+2)y^2$$

$$y' = \frac{dy}{dx}$$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\frac{dy}{y^2} = (x+2)dx$$

$$\frac{1}{y^2}dy = (x+2)dx$$

$$\int \frac{1}{y^2}dy = \int (x+2)dx$$

$$\int y^{-2} dy = \int (x+2)dx$$

$$\frac{y^{-1}}{-1} = \left[\frac{x^2}{2} + 2x\right] + c_1$$

$$\frac{y^{-1}}{-1} = \left[\frac{x^2}{2} + 2x\right] + c_1$$

$$-y^{-1} = \frac{x^2}{2} + 2x + c_1$$

$$y^{-1} = -\frac{x^2}{2} - 2x - c$$

$$\boxed{y = \frac{1}{-\frac{x^2}{2} - 2x - c}}$$

Q No1 :-

(b) Estimate the general solution of $Y' = (Y+9x)^2$

solution:-

$$y' = (y+9x)^2 \quad \text{---} \quad (1)$$

set $y+9x = u$

$$\frac{d}{dx}(u) = \frac{d}{dx}(y+9x)$$

$$\frac{d}{dx}(y+9x) = \frac{du}{dx}$$

$$= \frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9$$

so eq(1) becomes

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\frac{du}{u^2+9} = dx$$

$$\frac{1}{u^2+9} du = dx$$

$$\int \frac{1}{u^2+9} du = \int dx$$

$$\int \frac{1}{(3)^2+(u)^2} du = \int dx$$

$$\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + c$$

$$\frac{u}{3} = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$

$$\boxed{y = -9x + 3\tan(x + c)}$$

Q No 38:

(a) find the general solution of

$$4y'' - 20y - 25y = 0$$

solution:-

$$4y'' - 20y + 25y = 0$$

$$(4D^2 - 20D + 25)y = 0$$

the characteristics eq is

$$4D^2 - 20D + 25 = 0$$

$$4D^2 - 10D - 10D + 25 = 0$$

$$2D(2D-5) - 5(2D-5) = 0$$

$$(2D-5) = 0 \Rightarrow D = 5/2$$

$$D = 5/2, \frac{5}{2}$$

Since the roots are separate so general solution is

$$y = (c_1 + c_2 x) e^{5/2 x}$$

$$\Rightarrow 4y'' - 20y' + 25y = 0$$

$$(4D^2 - 20D + 25)y = 0$$

Now the characteristics eq is

$$4D^2 - 20D + 25 = 0$$

$$2D^2 - 10D - 10D + 25 = 0$$

$$2D(2D-5) - 5(2D-5) = 0$$

$$(2D-5)(2D-5) = 0$$

(i) $2D-5=0 \Rightarrow 2D=5$

$\therefore D = 5/2$

(ii) $2D-5=0 \Rightarrow 2D=5$

$\therefore D = 5/2$

Since the given roots are real and equal, so the general solution is

$$y = (c_1 + c_2 x)e^{mx}$$

$$\boxed{y = (c_1 + c_2 x)\, e^{5/2 x}}$$

Q No 2:- Estimate the general solution of $x^3dx+y^3dy=0$

Solutions-

$$x^3dx+y^3dy=0$$

$$Mdx+Ndy=0$$

$$M=x^3, N=y^3$$

$$\frac{\partial M}{\partial y}=0, \frac{\partial N}{\partial x}=0$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \text{ so exact}$$

$$U=\int Mdu+K(y)$$

$$U=\int x^3du+K(y)$$

$$U=\frac{x^4}{4}+K(y) \quad (1)$$

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} k(y)$$

$$\frac{\partial u}{\partial y} = \frac{d}{dy} k(y)$$

Since we know that

$$\frac{\partial u}{\partial y} = N = y^3$$

$$y^3 = \frac{d}{dy} k(y) \Rightarrow \int y^3$$

$$\Rightarrow k(y) = \frac{y^4}{4} + c_1 \text{ put in eq ①}$$

$$U = \frac{x^4}{4} + k(y)$$

$$U = \frac{x^4}{4} + \frac{y^4}{4} + c_1$$

$$c_2 = \frac{x^4}{4} + \frac{y^4}{4} + c_1$$

$$\Rightarrow c_2 - c_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\boxed{\Rightarrow c = \frac{x^4}{4} + \frac{y^4}{4} = U \text{ Ans}}$$

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QNO 3: Estimate general solution of

$$4y'' - 6y - 7y = 0$$

solutions-

$$4y'' - 6y - 7y = 0$$

Auxilary eq is

$$\cancel{4D^2}\ 4d - 6d - 7 = 0$$

$$4D^2 - 6D - 7 = 0$$

so by quadratic formulla

Here $a = 4$, $b = -6$, $c = -7$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

p(10)

$$D = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-7)}}{2(4)}$$

$$D = \frac{6 \pm \sqrt{36+112}}{8}$$

$$D = \frac{6 \pm \sqrt{148}}{8}$$

Taking 2 as common

$$D = \frac{3 \pm \sqrt{37}}{4}$$

so $D_1 = \frac{3+\sqrt{37}}{4}$

$$D_2 = \frac{3-\sqrt{37}}{4}$$

Roots are real, so

$$y = c_1 e^{D_2 t} + c_2^{D_2 t}$$

$$y = c_1 e^{\frac{3+\sqrt{37}}{4}} + c_2^{\frac{3-\sqrt{37}}{4}}$$ Ans!

End