difforom

(1)

Q NO :- 1.

Sol: the following objective type

(1) The order of a matrix AB is <u>$m \times n$</u>.

(2) The number of non-zero rows in Echelon form is. one.

(3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix the $a = ?$

$|B| = 1 \times a - 4 \times 2$

$|B| = a - 8$

$a = 8$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ the $|A| = ?$

$A = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$= -2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

(v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol: If each element of a principal diagonal.

The given matrix is a ~~diagonal~~ scalar matrix because the diagonal element are same and non-diagonal are zero.

(vi) Sol $\frac{dy}{dx} + 2xy = y$?

$\frac{dy}{dx} + 2xy = y.$

$\frac{dy}{dx} = y - 2xy.$

$\frac{dy}{dx} = y(1-2x)$

$y\,dy = (1-2x)\,dx.$

$\int y\,dy = \int (1-2x)\,dx.$

$\frac{y^2}{2} = x - \frac{2x^2}{2} + c.$

$y^2 = 2x - 4x^2 + c$

(vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2 \text{ is.}$$

order $= 1$

degree $= 3$.

(viii) The order and degree of differential equation.

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

order = Two

degree = one

(ix) The differential equation

$$2\frac{dy}{dx} + x^2y = 2x + 3,\ y(0) = 5 \text{ is?}$$

Sol:-

$$2y' + x^2y = x^3 + 3,\quad y(0) = 5.$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}.$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

d. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

Expand by C_1.

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b).$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b.$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2.$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2.$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$

part B

①

Q #2 Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

characteristic equ $\Rightarrow |A-\lambda I|=0 \rightarrow (A)$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant

$|A-\lambda I| = 0 \rightarrow (A)$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Expand by Row1.

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

$\rightarrow (B)$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1.

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)[(3-\lambda)(2-\lambda)-(-1)(-1)] + 1[(-1)(2-\lambda)-(-1)(-1)]$$
$$-1[(-1)(-1)-(-1)(3-\lambda)]].$$

$$= (3-\lambda)(6-\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda).$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \Rightarrow \text{eq (i)}$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by $C1$:

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0.$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$\Rightarrow \cancel{-\lambda^2+5\lambda-8\lambda+6}$$

$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$= \boxed{-\lambda^2 + 6\lambda - 8} \rightarrow (ii)$

$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$ Expand by C_1

$-[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$

$\Rightarrow -[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)$

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda.$

$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \rightarrow (ii)$

Put Eq (i), (ii) & (iii) in eq (B).

$= (2-\lambda)[-\lambda^3 + 8\lambda^2 - 18\lambda + 8] - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8$

$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2$
$+ 6\lambda - 8 - \lambda^2 + 16\lambda - 8.$

$\Rightarrow \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda$

$+ 6\lambda + 6\lambda + 16 - 16.$

$\Rightarrow \lambda^4 - 10\lambda^3 + 30\lambda^2 - 32\lambda = 0$

By synthetic division we get:-

$\lambda(\lambda - 2)(\lambda^2 - 8\lambda + 16) = 0$

$(\lambda = 0)$

$\lambda - 2 = 0 \Rightarrow \lambda = 2.$

$\lambda^2 - 8\lambda + 16 = 0.$

By factorization

$\lambda^2 - 8\lambda + 16$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0.$

$\lambda(\lambda - 4)(-4(\lambda - 4)) = 0.$

$(\lambda - 4)(\lambda - 4)$

$\lambda - 4 = 0, \quad \lambda - 4 = 0.$

$\lambda = 4, \quad \lambda = 4.$

$$\boxed{\lambda_1 = 0, \quad \lambda_2 = 2, \quad \lambda_3 = 4, \quad \lambda_4 = 4}$$

Q2:- Part A:-

Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c

Sol:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by C_1.

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$

$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$

Taking common

$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$\Rightarrow abc[bc(c-b) - ac(c+c) + ab(b-a)]$

Q.3)

$(x^2+3y^2)\,dx - 2xy\,dy = 0$.

$x=2,\ y=6$.

Sol:-

$(x^2+3y^2)\,dx - 2xy\,dy = 0$,

$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy$,

$\Rightarrow$

Dividing both sides by

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (i)$$

let $y = vx$

Divide by dx.

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \rightarrow (a).$$

put (a) in (i)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right.$$

$$v + x\frac{dv}{dx} = f$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right].$$

xing both side by 2.

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

multiplying both sides by $\frac{dx}{dv}$ we get.

$$2x\,dv = \frac{1+v^2}{v}dx.$$

multiplying by $\frac{v}{x(1+v^2)}$ on both sides

$$\frac{v}{1+v^2}dv = \frac{1}{x}dx.$$

Tak $\int$ on both sides.

$$\int\frac{2v}{1+v^2}dv = \int\frac{1}{x}dx + c.$$

Scanned with CamScanner

3) ①

$\ln|1+v^2| = \ln x + \ln c$.

Take "e" on b. sides.

$e^{\ln|1+v^2|} = e^{\ln|x c|}$

$1+v^2 = xc$

Put $v = y/x$

$1+(y/x)^2 = xc$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3 c \rightarrow (ii)$.

Put $x=2$, $y=6$ in (ii).

$(2)^2+(6)^2 = 8c$

$4+36 = 8c$

$\frac{40}{8} = \frac{8c}{8} \Rightarrow \boxed{c=5} \rightarrow$ (A)

so $x^2+y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x-1)$.

Taking "$\sqrt{\ }$" on b. sides.

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

$\boxed{y = \pm x\sqrt{5x-1}}$