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Q No. 1

Given data

coordinate of $P = (4, 1, 3)$

Coordinate of $Q = (1, 2, 4)$

Required

Distance b/w P and $Q = ?$

position of vector of point dividing PQ in ratio $1:3$

Solution

Coordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3k$

coordinate of $Q = (1, 2, 4)$

$OQ = (1i, 2j, 4k)$

formula = $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

put value.

$d = \sqrt{(1-4)^2 + (2-1) + (4-3)^2}$

$d = \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$d = \sqrt{9+1+1}$

$d = \sqrt{11}$ $\quad \boxed{d = 3.3167} \rightarrow ①$

Let M be the point which divide PQ in ratio 1:3 Then by ratio theorm

position vector of M $= \vec{OM}$

$$= \frac{3(4i+1j+3k)+(1)(i+2j+4k)}{1+3}$$

$$= \frac{(12i+3j+9k+i+2j+4k)}{4}$$

$$= \frac{13i+5j+13k}{4} \rightarrow ②$$

Hence ① ② ~~and ③~~ are required.

Q No 2

Evaluate $\int \frac{4x^3+10x+4}{2x^2+x}\,dx$

Solution

$$\int \frac{4x^3+10x+4}{2x^2+x} = \int \frac{2(2x^3+5x+2)}{x(2x+1}\,dx$$

Apply linearity

$$= 2\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$$

Now Solving

$$\int \frac{2x^3+5x+2}{x(2x+1}\,dx$$

perform polynomial long division

$$= \int \left(\frac{11x+4}{2x(2x+1)} + \frac{2x-1}{2}\right) dx$$

Apply linearity

$$= \frac{1}{2}\int \frac{11x+4}{x(2x+1}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

Now Solving

$$\int \frac{11x+4}{x(2x+1}\,dx$$

perform partial fraction decompasition

$$= \int \left(\frac{3}{2x+1} + \frac{4}{x}\right) dx$$

$= 3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$

Now Solving

$\int \frac{1}{2x+1}\,dx$

Substitute $u = 2x+1$ $\frac{du}{dx} = 2$ $dx = \frac{1}{2}du$

$= \frac{1}{2}\int \frac{1}{u}\,du$

Now Solveing

$\int \frac{1}{u}\,du$

This is Standard Integral

$= In(u)$

$\frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{In(u)}{2}$

put aganin $U = 2x+1$

$= \frac{In(2x+1)}{2}$

Now Solving

$\int \frac{1}{x}\,dx$

Use previous result

$= In(x)$

$3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$

$= \frac{3\ln(2x+1)}{2} + 4\ln(x)$

Now Solving

$$\int x\, dx$$

Apply power rule $\qquad \int x^n dx = \frac{x^{n+1}}{n+1}$ with $n=1$

$$= \frac{x^2}{2}$$

Now $\quad \int 1\, dx$

Apply constant rule

$$= x$$

$$\frac{1}{2}\int \frac{11x+4}{x(2x+1)} dx + \int x\, dx - \frac{1}{2}\int 1\, dx$$

$$= \frac{3\ln(2x+1)}{4} + 2(\ln(x)) + \frac{x^2}{2} - \frac{x}{2}$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x) + x^2 - x$$

Add constant to solution

$$= \frac{3\ln(|2x+1|)}{2} + 4\ln(|x|) + x^2 - x + C$$

$$= \frac{3\ln(|2x+1|)}{2} + 4\ln(|x|) + (x-1)x + C$$

$$= x^2 - x + \frac{3}{2}\ln|2x+1| + 4\ln|x| - \frac{3}{4} + C$$

Q No 3

A

$$\int_0^2 x^2 e^x dx$$

Solution $\int_0^2 x^2 e^x dx$

Apply Integration by parts

$$\int fg' = fg - \int f'g$$

$f = x^2$ $g' = e^x$

$f' = 2x,$ $g = e^x$

$$= x^2 e^x - \int 2xe^x dx$$

Now Solving

$$\int 2xe^x dx$$

$$= 2\int xe^x dx$$

Now Solving

$$\int xe^x dx$$

Integrate by parts

$$= xe^x - \int e^x dx$$

Now Solving

$$\int e^x dx$$

Apply exponential rule $\int a^x dx = \frac{a^x}{\ln(a)}$ with $a = e$:

$$= e^x$$

put in Solved Integral

$$xe^x - \int e^x dx$$

$$= xe^x - e^x$$

put

$$2\int xe^x dx$$

$$= 2xe^x - 2e^x$$

put $x^2e^x - \int 2xe^x dx$

$$= x^2e^x - 2xe^x + 2e^x$$

Add constant

$$= x^2e^x - 2xe^x + 2e^x + c$$

$= \cancel{x^2}e$

$$x^2e^x - 2xe^x + 2e^x + c$$

$$= (x^2 - 2x + 2)e^x + c$$

$\int f(x)dx = f(x) =$

$\int_0^2 f(x)dx =$ $(x^2 - 2x + 2)e^x + c \Rightarrow 2^2e^2 - 2(2) + 2e^2 - (0 - 0 + 2e^2)$

$$= (4e^2 - 4e^2 + 2e^2 - 2)$$

$$= 2e^2 - 2$$

$$\boxed{12.78}$$

Q3
(B) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

Solution $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}} dx \rightarrow (1)$

frist find Integration

Ley $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2dy = \frac{1}{\sqrt{x}} dx$ put in (1)

$\int \sin(y)(2dy) = 2\int \sin(y)\, dy$

$= 2(-\cos y)$

$= -2\cos y$

put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

put limit

$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$

$= -2\cos\sqrt{2} + 2\cos(1)$

$\boxed{= 0.7681}$

Q No 4

verify that

$$v(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the Three-dimensional Laplace's equation

The laplace equation in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow A$$

So

$$U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$U(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$= \frac{\partial u}{\partial x} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$= \frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)$$

$$= \frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2}(x^2+y^2+z^2)^{-3/2} \rightarrow ①$$

Now

$$= \frac{\partial v}{\partial y} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$= \frac{\partial v}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2) \right.$$

$$= \frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \to (2)$$

$$= \frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$= \frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$= \boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2}} \quad (3)$$

= putting (1) (2) and (3) in A

$$= 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2) + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$$
$$- (x^2+y^2+z^2)$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right)$$

$$= (x^2+y^2+z^2)^{-5/2}\left(\cancel{3x^2} - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 + x^2 + y^2 + z^2\right)$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So The given $u(x,y,z)$ is solution of Laplace equation.