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Subject Probability & statistics

SEMESTER 4Th

DEPARTMENT BSCS

Q:1

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

Let

$A = \{$ The sum is 7 $\}$

$B = \{$ The sum is even $\}$

C = { The sum is greater than 8 }

D = { The two dise had the same outcomes }

Now

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1)$
$(3,3), (3,5), (4,2), (4,4), (4,6), (5,1)$
$(5,3), (5,5), (6,2), (6,4), (6,6)\}$

$C = \{(3,6), (4,5), (4,6), (5,4), (5,5)$
$(5,6), (6,3), (6,4), (6,5)$
$(6,6)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5)$
$(6,6)\}$

$A \cap B = \{\ \}$ OR ϕ

$A \cap C = \{\ \}$ OR ϕ

$A \cap D = \{\ \}$ OR ϕ.

$P(A) = 6/36$, $P(B) = 18/36$

$P(C) = 10/36$, $P(D) = 6/36$.

Hence :-

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0 \times 18/36$$

$$P(A/B) = 0$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = 0 \times 10/36$$

$$P(A/C) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times 6/36$$

$$P(A/D) = 0$$

Q.2

When we are rolling two dice, there are 36 different combination. There are 15 possibilities less than 7: (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2) (3,3), (4,1), (4,2), (5,1), 5.

The probability of getting less than 7 is

$$15/36 = 5/12$$

There are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4) (4,3), (5,2), (6,1) which is given a probability of.

$$6/36 = 1/6$$

This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting more than 7.

Q3

$p = 2/3$

$n = 8$

$q = 1 - p$

$= 1 - 2/3$

$q = 1/3$

i) $P(X=4)$:-

$$\binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= 0.1707$$

ii) $P(x \geq 4)$

$$1 - P(x<4)$$

$$= 1 - \sum_{x=0}^{3}\binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-x}$$

$$1-\left[\left(\frac{1}{3}\right)+8\left(\frac{2}{3}\right)\left(\frac{1}{7}\right)^7+28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6+56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$$

$$=1-\frac{1}{6561}\left[1+16+112+448\right]$$

$$=1-\frac{577}{6561}$$

$$=\frac{6561-577}{6561}$$

$$=\frac{5984}{6561}$$

$$=0.9121$$

iii) $P(3 \le x \le 6)$

$$\sum_{x=3}^{6}\binom{8}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{8-4}$$

$$=\binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5+\binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4+\binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3+\binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$$

$$=\frac{8}{(3)^8}\left[56+140+224+224\right]$$

$$=\frac{8\times 644}{6561} \Rightarrow \frac{5152}{6561}$$

$$=0.7852$$

Q4.

Since the C_is form a partition of the sample space, we can apply the law of total probability for $A \cap B$

$$\star \quad P(A\cap B) = \sum_{i=1}^{m} P(A\cap B / C_i)\, P(C_i)$$

$$P(A\cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B/C_i)\, P(C_i)$$

A and B are conditionally independent.

$$\star \quad P(A\cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B)\, P(C_i)$$

$\star$ B is independent of all C_i's

$$\star \quad P(A\cap B) = P(B) \sum_{i=2}^{m} P(A/C_i)\, P(C_i)$$

$$P(A\cap B) = P(B)\, P(A)$$

Hence A & B are independent.

Q5

The probability function for a binomial random variable is

$$b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability of having x successess in a series of n independent trials when the probability of success in any one of the trials is p.
if X is a random variable with this probability distribution.

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$x=0$ term vanishes.

Let $y = x-1$ & $m = n-1$

Adding $x = y+1$ & $n = m+1$ into

the last sum (and using the fact that the limits $x=1$ and $x=n$ correspond to $y=0$ and $y=n-1=m$ respectively.

$$E(X) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

The binomial theorem says:

$$(a+b)^{m} = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y}$$

Putting $a=p$ & $b=1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y} = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y} =$$

$$(a+b)^{m} = (p+1-p)^{m} = 1$$

so that

$$E(X) = np$$

Similarly using $y = x-2$ & $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p + (1-p))^m$$

$$= n(n-1)p^2$$

So variance of x is.

$$E(x^2) - E(x)^2 = E(x(x-1)) + E(x) - E(x)^2 =$$

$$n(n-1)p^2 + np - (np)^2$$

$$\boxed{= np(1-p)}$$ Ans.

Q6

Bi-nominal Distribution :-

Many experiments consist of repeated independent trials, each trial having two possible outcomes. e.g.
the two possible outcomes of a trial may head and tail, success and Failure.

$$P(X=x)\ f(x) = {}^{n}C_{x}\, P^{x} q^{n-x}$$

Bi-nominal Frequency :-

if the bi-nomial probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bi-nomial frequency distribution.

$$N\binom{n}{x} P^{x} q^{n-x}$$

Q 7

Coppicient of variation:-

For Data set A:-

$$CV = \frac{\sigma}{\mu}$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

For Data set B:-

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

For Data set C:-

$$CV = \frac{5}{50} \times 100$$

$$CV = 10$$

For Data set D :-

$$CV = \frac{\sigma}{\mu}$$

$$CV = 15/25 \times 100$$

$$CV = 60$$