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Subject

calculus

Assignment

sessional

Submitted to

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Q NO 1-

Ans:- Given $\int_0^{\pi/4} (1 - \sin t)^{3/2} \cos 2t \, dt$

Solution

Integration by part

$$(1 - \sin t)^{3/2} (\cos 2t) = \int (1 - \sin t)^{3/2} \cos 2t \int (\cos 2t) dt$$
$$= 3/2 \left((1 - \sin t)^{1/2} \cos t - \frac{\sin 2t}{2} \right)$$

$$\left[(1 - \sin t)^{3/2} (\cos 2t) \right]_0^{\pi/4} = \frac{6}{4} \int_0^{\pi/4} (1 - \sin t) \cos t \sin^2 t$$

Now let

$$u = \sin t$$

$$d/dt u = \cos t$$

$$du = \cos t \, dt$$

$$(1 - \sin t)^{3/2} (\cos 2t) = \frac{6}{4} \int_0^{\pi/4} (1 - u)^{1/2} u^2 du$$

Now let

$$t = 1 - u$$

$$\frac{dt}{du} = -1$$

$$dt = -du$$

$$dt = -du$$

$$= 6/4 \int_0^{\pi/4} (1 - t)^2 t^{1/2} - dt$$

$$= -6/4 \int_0^{\pi/4} (1 - 2t + t^2) (t^{1/2}) dt$$

applying integral

$$= \frac{6}{4} \int_0^{\pi/4} (t^{1/2} - 2t^{3/2} + t^{5/2}) dt$$

applying Integral

$$= \frac{-6}{4} \left[\frac{t^{5/2+1}}{\frac{5}{2}+1} - \frac{2t^{3/2+1}}{\frac{3}{2}+1} + \frac{t^{1/2+1}}{\frac{1}{2}+1} \right]_0^{\pi/4}$$

$$= \frac{-6}{4} \left[\frac{2}{7} (t)^{7/2} - \frac{4}{5} t^{5/2} + \frac{2}{3} t^{3/2} \right]_0^{\pi/4}$$

Replace the values

$$= \frac{-6}{4} \left[\frac{2}{7} (\sin t)^{7/2} \Big|_0^{\pi/4} - \frac{4}{5} (\sin t)^{5/2} \Big|_0^{\pi/4} + \frac{2}{3} (\sin t)^{3/2} \Big|_0^{\pi/4} \right]$$

$$= \frac{-6}{4} \left(\frac{2}{7} (\sin 45)^{7/2} - \frac{4}{5} (\sin 45)^{5/2} + \frac{2}{3} (\sin 45)^{3/2} \right)$$

$$= \frac{-6}{4} (0.28 + 0.085 - 0.33)$$

$$= \frac{-6}{4} (0.035)$$

$$= 0.0525 \text{ Ans.}$$

Q 2i- $\int (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy$
 Sol:- $\int (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy$

By Using Substitution method

let $x = 4y - y^2 + 4y^3 + 1$

$$\begin{aligned} \frac{dx}{dy} &= \frac{d}{dy} (4y - y^2 + 4y^3 + 1) \\ &= \frac{d}{dy} 4y - \frac{d}{dy} y^2 + \frac{d}{dy} 4y^3 + \frac{d}{dy} 1 \\ &= 4 - 2y + 12y^2 \end{aligned}$$

$$dx = (4 - 2y + 12y^2) dy$$

$$dx = (12y^2 - 2y + 4) dy$$

$$\int x^{-2/3} dx$$

$$\frac{x^{-2/3 + 1} + C}{-2/3 + 1}$$

$$\frac{x^{1/3}}{1/3}$$

$$3x^{1/3} + C$$

Now put the value of x

$$3(4y - y^2 + 4y^3 + 1)^{1/3} + C$$

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Putting the limits

$$\begin{aligned} & 3 \left(4(1) - (1)^2 + 4(1)^3 + 1 \right)^{1/3} + c \\ & 3 \left(4 - 1 + 4 + 1 \right)^{1/3} + c \\ & 3 \left(4 + 4 \right)^{1/3} + c \\ & 3 \left(8 \right)^{1/3} + c \\ & 3 \left(2^3 \right)^{1/3} \\ & 3 (2) \\ & 6 \end{aligned}$$