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Exam Type →	Improvement For GPA Summer
Paper	Differential Equation
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Q: 1

Solve the Initial Value Problem

$$\frac{dy}{dt} = e^{y-t} \sec y \, (1+t^2) \qquad y(0)=0$$

$$\frac{dy}{dt} = e^{y} \cdot e^{-t} \sec y \, (1+t^2)$$

Separating variables

$$dy = e^{y} \cdot e^{-t} \sec y \, (1+t^2)\, dt$$

$$\frac{dy}{e^{y} \sec y} = e^{-t} (1+t^2)\, dt$$

$$e^{-y} \cos y \, dy = (e^{-t} + t^2 e^{-t})\, dt$$

$$\int e^{-y} \cos y \, dy = \int e^{-t} dt + \int t^2 e^{-t} dt \longrightarrow (1)$$

$$I = \int e^{-y} \cdot \cos y \, dy$$

$$I = e^{-y} \int \cos y \, dy - \int \left(\frac{d}{dy} e^{-y} \int \cos y \, dy \right) dy$$

$$I = e^{-y} \sin y - \int - e^{-y} \sin y \, dy$$

$$I = e^{-y} \sin y + \int e^{-y} \sin y \, dy$$

$$I = e^{-y} \sin y + \left\{ e^{-y} (-\cos y) - \int -e^{-y} (-\cos y) dy \right\}$$

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$$I = e^{-y}\sin y + \left\{-e^{-y}\cos y - \int e^{-y}\cos y\, dy\right\}$$

$$I = e^{-y}\sin y - e^{-y}\cos y - \int e^{-y}\cos y\, dy$$

$$I = e^{-y}\sin y - e^{-y}\cos y - I$$

$$I + I = e^{-y}\sin y - e^{-y}\cos y$$

$$2I = e^{-y}(\sin y - \cos y)$$

$$\boxed{I = \frac{e^{-y}}{2}(\sin y - \cos y)}$$

$$\int e^{-t}\, dt = \frac{e^{-t}}{-1} + C = -e^{-t} + C$$

$$\int t^2 e^{-t}\, dt = t^2 \frac{e^{-t}}{-1} - \int (2t)\frac{e^{-t}}{-1}\, dt$$

$$= -t^2 e^{-t} + 2\int t\cdot e^{-t}\, dt$$

$$= -t^2 \cdot e^{-t} + 2\left\{t\frac{e^{-t}}{-1} - \int 1\cdot\frac{e^{-t}}{-1}\, dt\right\}$$

$$= -t^2 e^{-t} + 2\left\{-te^{-t} + \int e^{-t}\, dt\right\}$$

$$= -t^2 e^{-t} + 2\left\{-te^{-t} + \frac{e^{-t}}{-1}\right\}$$

$$= -t^2 e^{-t} - 2te^{-t} - 2e^{-t}$$

So equation (1) becomes;

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -e^{-t} - t^2e^{-t} - 2te^{-t} - 2e^{-t} + c$$

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -3e^{-t} - t^2e^{-t} - 2te^{-t} + c \quad \text{(ii)}$$

Put $t=0$, $y=0$ in eq (ii)

$$\frac{e^{0}}{2}(\sin 0 - \cos 0) = -3e^{0} - 0 - 0 + c$$

$$\frac{1}{2}(0-1) = -3 + c$$

$$-\frac{1}{2} + 3 = c$$

$c = 5/2$ put in equation (ii)

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -3e^{-t} - t^2e^{-t} - 2te^{-t} + \frac{5}{2}$$

Multiplying "2" on both sides

$$e^{-y}\cdot(\sin y - \cos y) = -6e^{-t} - 2t^2e^{-t} - 4te^{-t} + 5$$

Ans:

P.T.O →

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Comment:

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Is this question will be like

$$\frac{dy}{dt} = e^{y-t} \sec y (1+t^2)$$

· Instead of

$$\frac{dy}{dx} = e^{y-t} \sec y (1+t^2) \text{ ?}$$

Q.1 Complete

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Q:2

Solve

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx=\left(\sqrt{x+y}-\sqrt{x-y}\right)dy$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$$

$$\frac{dy}{dx}=\frac{\sqrt{x}\sqrt{1+y/x}+\sqrt{x}\sqrt{(1-y/x)}}{\sqrt{x}\sqrt{1+y/x}-\sqrt{x}\sqrt{1-y/x}}$$

$$\frac{dy}{dx}=\frac{\sqrt{x}\left[\sqrt{1+y/x}+\sqrt{1-y/x}\right]}{\sqrt{x}\left[\sqrt{1+\frac{y}{x}}-\sqrt{1-y/x}\right]}$$

Let $\frac{y}{x}=V$

$$y=Vx$$

$$\frac{dy}{dx}=V.1+x\frac{dV}{dx}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} - \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} - \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{(\sqrt{1+v})^2 - (\sqrt{1-v})^2}{(1+\cancel{v}) + (1-\cancel{v}) - 2\sqrt{1-v^2}}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{1} + v - \cancel{1} + v}{2 - 2\sqrt{1-v^2}}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}v}{\cancel{2}(1-\sqrt{1-v^2})}$$

$$v + x\frac{dv}{dx} = \frac{v}{1-\sqrt{1-v^2}}$$

(3)

$$x\frac{dv}{dx} = \frac{v}{1-\sqrt{1-v^2}} - v$$

$$x\frac{dv}{dx} = \frac{v - v\left(1-\sqrt{1-v^2}\right)}{1-\sqrt{1-v^2}}$$

$$x\frac{dv}{dx} = \frac{\cancel{v} - \cancel{v} + \sqrt{1-v^2}}{1-\sqrt{1-v^2}}$$

$$x\,dv = \frac{\sqrt{1-v^2}}{1-\sqrt{1-v^2}}\,dx$$

$$\frac{1-\sqrt{1-v^2}}{\sqrt{1-v^2}}\,dv = \frac{dx}{x}$$

$$\left(\frac{1}{\sqrt{1-v^2}} - 1\right)dv = \frac{1}{x}\,dx$$

$$\int \frac{1}{\sqrt{1-v^2}}\,dv - \int dv = \int \frac{1}{x}\,dx$$

$$\sin^{-1} v - v = \ln x + C$$

$$\sin^{-1}\frac{y}{x} - \frac{y}{x} = \ln x + C \longrightarrow (i)$$

Q: 2 Complete:

(9)

Q.No (3)

Solve

$$(D^4 + D^2) y = 3x^2 + 4 \sin x - 2 \cos x$$

The characteristic equation is

$$D^4 + D^2 = 0$$

$$D^2 (D^2 + 1) = 0$$

$$D^2 = 0 \qquad D^2 + 1 = 0$$

$$\sqrt{D^2} = \sqrt{0} \qquad D^2 = -1$$

$$D = 0 \qquad D^2 = \pm\sqrt{-1}$$

$$D = \pm i$$

$$D = 0, i, -i$$

$$y_p = c_1 e^{0.x} + e^{0.x}(c_2 \cos x + c_3 \sin x) + e^{0.x}(c_4 \cos(-x) + c_5 - \sin(-x))$$

$$y_c = c_1 + c_2 \cos x + c_3 \sin x + c_4 \cos x - c_5 - \sin x$$

For Particular Solution

$$y_p = \frac{1}{(D+0)(D-i)(D+i)} (3x^2 + 4 \sin x - 2 \cos x)$$

$$y_p = \frac{1}{(D-i)(D+i)} \frac{1}{D}(3x^2 + 4\sin x - 2\cos x)$$

$$y_p = \frac{1}{(D-i)(D+i)}\left[x^3 - 4\cos x - 2\sin x\right]$$

$$y_p = \frac{1}{2i}\left[\frac{1}{D-i} - \frac{1}{D+i}\right](x^3 - 4\cos x - 2\sin x)$$

$$y_p = \frac{1}{2i}\left[\frac{1}{D-i}(x^3 - 4\cos x) - 2\sin x)\right] - \frac{1}{D+i}(x^3 - 4\cos x - 2\sin x) \longrightarrow (1)$$

$$\frac{1}{D-i}(4\cos x) = 4\frac{1}{D-i}(\cos x)$$

$$= 4e^{ix}\int e^{-ix}\cos x\,dx$$

$$= 4e^{ix}\int e^{-ix}\left(\frac{e^{ix}+e^{-ix}}{2i}\right)dx$$

$$= \frac{4e^{ix}}{2i}\int e^{-ix}(e^{ix} + e^{-ix})\,dx$$

$$= \frac{2e^{ix}}{i}\int (e^{0} + e^{-2ix})\,dx$$

$$= \frac{2e^{ix}}{i}\left[x - \frac{e^{-2ix}}{2i}\right]$$

(11)

$$\frac{1}{D-i}\left[-2\sin x\right] = -2\frac{1}{D-i}(\sin x)$$

$$= -2e^{ix}\int e^{-ix}\sin x\,dx$$

$$= -2e^{ix}\int e^{-ix}\left(\frac{e^{ix}-e^{-ix}}{2i}\right)dx$$

$$= -\frac{2e^{ix}}{2i}\int (e^{0}-e^{-2ix})\,dx$$

$$= \frac{-e^{ix}}{i}\left[x-\frac{e^{-2ix}}{-2}\right]$$

$$= \frac{e^{-ix}}{i}\left(x+\frac{e^{2ix}}{2}\right)$$

replace i by $-i$

$$\frac{1}{D+ia}\left[4\cos x\right) = \frac{2e^{-ix}}{-i}\left[x+\frac{e^{2ix}}{2i}\right]$$

$$\frac{1}{D+ia}\left[-2\sin x\right] = -2\frac{e^{ix}}{-i}\left(x+\frac{e^{-2ix}}{-2}\right)$$

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$$y_p = \frac{1}{2i}\left[x^4 + \frac{2e^{ix}}{i}\left[x - \frac{e^{-2ix}}{2i}\right] + \left(\frac{e^{-ix}}{i}\left(\frac{x + e^{2ix}}{2}\right)\right)\right.$$

$$+ \frac{1}{2i}\left[x^4 + \frac{2e^{-ix}}{-i}\left(x + \frac{e^{2ix}}{2i}\right)\right.$$

$$- 2\frac{e^{ix}}{-i}\left(x + \frac{e^{-2ix}}{-2}\right)$$

So the Solution is

$$y = y_c + y_p$$

Q:3 Complete