Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

FINAL – Term Examination summer 2020
Date:23/9/2020

Course Code: MTH 102		**Course Title:**	Calculus and analytic geometry
Prerequisite:		**Instructor:**	HIMAYATULLAH
Module: 3	**Program:** BEE	**Total Marks:** 50	**Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	a	. **Estimate** $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$	Marks 7 PLO2 C2
	b	**Estimate** $\int_0^1 x^3(1+x^4)^3 dx$ using substitution method.	Marks 7 PLO2 C2
Q2		The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. **Apply** the integration find the volume of solid.	Marks 9 PLO1 C3
Q3		If A= $2i-4j+\sqrt{5}k$, *and* $B=-2i+4j-\sqrt{5}k$ then **illustrate** (i) the scalar component of B in the direction of A (ii) the vector $proje_A\, B$	Marks 9 PLO1 C3
Q4		**Apply** the Fubini's theorem calculate $\iint_R f(x,y)dA$ *for* $f(x,y)=1-6x^2y$ *and* $R; o \le x \le 2,\ -1 \le y \le 1$	Marks 9 PLO1 C3
Q5	(a)	Find the maxima, minima, of the curve $y=-2x^3+6x^2-3$.	Marks 5 PLO1 C3
	(b)	**Change** into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$	Marks 4 PLO1 C3

			PLO2 C2

Q Name: Naveed Ali

ID No. 16753.

Q1 part (A)

Estimate $\int Q \sqrt[4]{1-Q^2}\, dQ$

Sol:

$$\int Q \sqrt[4]{1-Q^2}\, dQ$$

Let $u = 1-Q^2$

$$du = 0 - 2Q\, dQ$$

$$du = -2Q\, dQ$$

$$\frac{du}{-2} = Q\, dQ$$

$$\int \sqrt[4]{1-Q^2}\; Q\, dQ$$

$$\int \sqrt[4]{u} - du/2$$

$$= -1/2 \int (u)^{1/4}\, du$$

$$= -1/2 \left(\frac{u^{1/4+1}}{1/4+1} \right) + c$$

$$= -1/2 \left(\frac{u^{5/4}}{5/4} \right) + c$$

$$= -1/2 \times \frac{u^{5/4}}{5} \times 4 + c$$

$= \frac{-2 \times 4}{5}{}^{5/4} + c$

replace $u = 1 - \theta^2$

$= -2/5 \, (-\theta^2 + 1)^{5/4}$

$= -2/5 \, (-\theta^2 + 1)^{5/4} + c$

Ans

— x — x — x — x — x — x —

2.

Q1: Part B.

Estimate $\int_0^1 x^3(1+x^4)^3\,dx$ using substitution method.

Sol: Let $u = 1+x^4$

$du = 0+4x^3\,dx$

$du = 4x^3\,dx$

$du/4 = x^3\,dx$

Substitute.

$$\int_0^1 (1+x^4)^3 x^3\,dx$$

$$\int_0^1 (u)^3\,du/4 = 1/4 \int_0^1 u^3\,du$$

$$= 1/4\,(u)^{3+1}/_{3+1} + C$$

$$1/4 \int_0^1 u^3\,du$$

$$1/4\left[\frac{u^4}{4}\Big|_0^1\right] + C$$

replace $u = 1+x^4$

$$1/4\left[\frac{(1+x^4)^4}{4}\Big|_0^1\right] + C$$

$$\frac{1}{16}\left[(1+1^{2})^4 - [1+(0)^4]^4\right]+c$$

$$\frac{1}{16}\left[(2)^4 - (1+0)\right]+c$$

$$\frac{1}{16}\left[16-1\right]+c$$

$$\int_0^1 x^3(1+x^4)\,dx =$$

$$\left(\frac{15}{16}+c\right)$$ Ans.

4.

Q5 (A) Find the maxima, minima of the curve $y = -2x^3 + 6x^2 - 3$.

Sol:

$y = -2x^3 + 6x^2 - 3$

1st Find $f'(x)$ & $f''(x)$

$f'(x) = dy/dx = dy/dx\ [-2x^3 + 6x^2 - 3)$

$= -2\, d/dx\ x^3 + 6\, d/dx\ x^2 - d/dx\ 3$

$= -2(3x^2) + 6\ (2x) - 0$

$= -6x^2 + 12x - 0$

$f'(x) = -6x^2 + 12x$.

Now

$f'(x) = d/dx\ (-6x^2 + 12x)$

$f''(x) = -12x + 12$.

for maxima & minima

$f'(x) = 0$

$-6x^2 + 12x = 0$

$-6\ [-x^2 + 2x) = 0$

$-x^2 + 2x = 0$

$x\ [-x + 2) = 0$

$-x + 2 = 0$

$\boxed{x = 2}$

Put $x=2$ in $f''(x)$

$f''(x) = (-12x + 12)$

put $x=2$

~~$f''(x)$~~

$f''(2) = (-12(2) + 12)$

$= 24 + 12$

$f''(2) = -12$

So.

$f''(x) < 0$

So function

$y = -2x^3 + 6x^2 - 3$ is mimime

at $x = 2$..

— x — x — x — x — x —

Part B.

Q5. chang into a spherd coordinate equation
(b) for the sphere $x^2 + y^2 + (z-1)^2 = 1$

Sol:- $x^2+y^2 + (z-1)^2 = 1$

$(\rho \sin\phi \cos\theta)^2 + (\rho \sin\phi \sin\theta)^2$

$+ (\rho \cos\phi - 1)^2 = 1$

$\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta$

$+ \rho^2 \cos^2\phi + 1 - 2\rho \cos\phi = 1$

$\rho^2 \sin^2\phi (\cos^2\theta + \sin^2\theta)$

$+ \rho^2 \cos^2\phi + 1 - 2\rho \cos\phi = 1$

$\rho^2 (\sin^2\phi) + \rho^2 \cos^2\phi + 2\rho \cos\phi = 1-1$

$\rho^2 (\sin^2\phi + \cos^2\phi) - 2\rho \cos\phi = 0$

$\rho^2 = 2\rho \cos\phi = 2\cos\phi.$

7.8

— x — x — x — x — x — x —

Q2. The region between the curve $y = \sqrt{x}$ $0 \le x < 4$, & the x-axis is resolved about the x-axis to generate a solid. Apply the integration find volume of solid.

Sol::

$y = \sqrt{x}$

$0 \le x \le 4 \Rightarrow a \le x \le b$

As

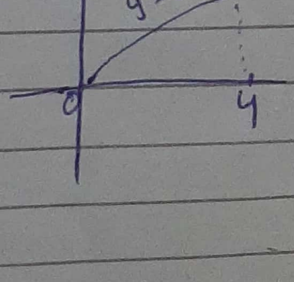
$U = \int_a^b \pi y^2 \, dx.$

$V \int_0^4 \pi (\sqrt{x})^2 \, dx.$

$V = \pi \int_0^4 x \, dx = \pi \, \frac{x^2}{2} \Big|_0^4$

$V = \pi/2 \, [(4)^2 - 0)$

$\boxed{V = 8\pi}$ Ans.



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Q3: If $A = 2i - 4j + \sqrt{5}k$, & $B = -2i + 4j - \sqrt{5}k$ then illustrate (i) scalar component B in the direction of A. (ii) the vector $proj_A B$

Sol:

$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$B \cdot A = -4i - 16j - 5k$

$\boxed{BA = -25}$

$A \cdot A = (2i - 4j + 5k) \cdot (2i - 4j + \sqrt{5}k)$

$= 4 + 16 + 5$

$\boxed{A \cdot A = 25}$

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

$$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$$

$$= (-1\ (2i - 4j + \sqrt{5}k)$$

$$\boxed{= -2i + 4j - \sqrt{5}k}$$

Ans 9

Q4. Apply the fubini theorem calcute $\iint f(x,y)\,dA$ for $(x,y) = 1, 6x^2$ & $R \cdot 0 \le x \le 2 \cdot\cdot -1 \le y \le 1$.

Sol:-

fubini theorem.

$$= \iint f(x,y)\,dA.$$

$$= \int_0^2\int_{-1}^1 f(x,y)\,dA.$$

$$f(x,y) = 1 - 6x^2y$$

$$= \int_0^2 \int_{-1}^1 (1 - 6x^2y)\,dy\,dx.$$

$$= \int_0^2 \left(\int_{-1}^1 dy - 6x^2 \int_{-1}^1 y\,dy \right) dx.$$

$$= \int_0^2 \left(y\Big|_{-1}^1 - \frac{6x^2y^2}{2}\Big|_{-1}^1 \right) dx.$$

$$= \int_0^2 (1+1 - 3x^2 - 3x^2)\,dx$$

$$= \int_0^2 (2 - 6x^2)\,dx$$

$$= 2\int_0^2 dx - 6\int_0^2 x^2\,dx.$$

$$2x\Big|_0^2 - \cancel{6}^{2}\,\frac{x^3}{\cancel{3}}\Big|_0^2$$

$$= 4 - 2(8)$$

$$= 4 - 16$$

$$\boxed{= -12}$$ Ans

11.