Name------------------- Hidayatullah

ID------------------------ 16495

Section---------------- (B)

Department--------- BS Software Engineering

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Subject---------------- Discrete structure

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Teacher--------------- Muhammad Ibrar Khan

Note: Attempt all questions. Use examples and diagrams where necessary.

Q.1

a) **Explain the concept of Biconditional statement.**

Answer (a): Biconditional statement

A biconditional statement is defined to be true whenever both parts have the same truth value. The biconditional operator is denoted by a double-headed arrow ↔. The biconditional p↔q represents "p if and only if q," where p is a hypothesis and q is a conclusion. The following is a truth table for biconditional p↔q.

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

In the truth table above, p↔q is true when p and q have the same truth values, (i.e., when either both are true or both are false.) Now that the biconditional has been defined, we can look at a modified version of Example 1.

Example 1:

Given:	**p: A polygon is a triangle.**
	q: A polygon has exactly 3 sides.
Problem:	**What does the statement p↔q represent?**
Solution:	**The statement p↔q represents the sentence, "A polygon is a triangle if and only if it has exactly 3 sides."**

Note that in the biconditional above, the hypothesis is: "A polygon is a triangle" and the conclusion is: "It has exactly 3 sides." It is helpful to think of the biconditional as a conditional statement that is true in both directions.

Remember that a conditional statement has a one-way arrow (→) and a biconditional statement has a two-way arrow (↔). We can use an image of a one-way street to help us remember the symbolic form of a conditional statement, and an image of a two-way street to help us remember the symbolic form of a biconditional statement.

Let's look at more examples of the biconditional.

Example 2:

Given:	**a: x + 2 = 7**

	b: $x = 5$
Problem:	**Write $a \leftrightarrow b$ as a sentence. Then determine its truth values $a \leftrightarrow b$.**

Solution: The biconditonal $a \leftrightarrow b$ represents the sentence: "$x + 2 = 7$ if and only if $x = 5$." When $x = 5$, both a and b are true. When $x \neq 5$, both a and b are false. A biconditional statement is defined to be true whenever both parts have the same truth

value. Accordingly, the truth values of $a \leftrightarrow b$ are listed in the table below.

a	**b**	**$a \leftrightarrow b$**
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

b) Let p, q, and r represent the following statements:

p: Sam had pizza last night.
q: Chris finished her homework.
r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.
ii. Pat watched the news this morning iff Sam did not have pizza last night.
iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.
iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

Answer (B)

i. **$p \leftrightarrow q$**
ii. **$r \leftrightarrow \sim p$**
iii. **$(r \leftrightarrow q)\wedge(\sim p)$**
iv. **$r \wedge p \wedge q$**

Q.2

a) Lets p, q, r represent the following statements:
p: it is hot today.
q: it is sunny
r: it is raining
Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$
ii. $p \leftrightarrow (q \wedge r)$
iii. $p \leftrightarrow (q \vee r)$
iv. $r \leftrightarrow (p \vee q)$

Answers:

i. **It is sunny if and only if it is hot today**
ii. **It is hot today iff it is sunny and it is raining**
iii. **It is hot today iff it is sunny or it is raining**
iv. **It is raining iff it is hot today or it is sunny**

c) **Q.3**
d) **Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments. (Note: Examples and truth table should not belongs to your book or slides)**

Answer: **Argument**

Argument is a list of statements (premises or assumptions or hypotheses) followed by a statement (conclusion)

P1 Premise

P2 Premise

. . . .

Pn Premise

∴ C Conclusion

For example, given the premises:

- **"if it is cloudy outside, then it will rain"**
- **"it is cloudy outside"**

a conclusion might be "it will rain". Intuitively, this seems valid.

Or

An argument is a sequence of statements. All statements except the final one are called premises (or assumptions or hypotheses). The final statement is called the conclusion.

• An argument is considered valid if from the truth of all premises, the conclusion must also be true. • The conclusion is said to be inferred or deduced from the truth of the premises Arguments

• Test to determine the validity of the argument:

– Identify the premises and conclusion of the argument

– Construct the truth table for all premises and the conclusion

– Find critical rows in which all the premises are true

– If the conclusion is true in all critical rows then the argument is valid, otherwise it is invalid

• Example of valid argument form:

– Premises: $p \vee (q \vee r)$ and $\sim r$, conclusion: $p \vee q$ • Example of invalid argument form: – Premises: p ◊ q ∨ ~r and q ◊ p ∧ r, conclusion: p ◊ r

Differentiate Valid and Invalid argument

Valid & Invalid Argument

• Argument is valid if the conclusion is true when all the premises are true or if conjunction of its premises imply conclusion.

$(P1 \wedge P2 \wedge P3 \wedge \ldots \wedge Pn) \rightarrow C$ is a tautology.

• Argument is invalid if the conclusion is false when all the premises are true or if conjunction of its premises does not imply conclusion.

$(P1 \wedge P2 \wedge P3 \wedge \ldots \wedge Pn) \rightarrow C$ is a Contradiction.

• A valid argument may have:

• true premises and a true conclusion

• or false premises and a false conclusion

• or false premises and a true conclusion

• but it cannot have all true premises and yet a false conclusion

• Arguments may either valid or invalid; and statements may either true or false

Or

Valid: **an argument is valid if and only if it is necessary that if all of the premises are true, then the conclusion is true; if all the premises are true, then the conclusion must be true; it is impossible that all the premises are true and the conclusion is false.**

Invalid: **an argument that is not valid. We can test for invalidity by assuming that all the premises are true and seeing whether it is still possible for the conclusion to be false. If this is possible, the argument is invalid.**

Valid and Invalid Examples:

#1

Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.

Kanoe lives on the island of Oahu.

Therefore, Kanoe lives in the city Honolulu, HI.

#2

Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.

Kanoe does not live on the island of Oahu.

Therefore, Kanoe does not live in the city Honolulu, HI.

#3

Anyone who lives in the city Honolulu, HI also lives on the island of Oahu.
Kanoe does not live in the city Honolulu, HI.
Therefore, Kanoe does not live on the island of Oahu.

#5
All crows are black.
John is black.
Therefore, John is a crow.

#6
Only crows are black.
John is black.
Therefore, John is a crow.

Answers:
#1 Invalid
#2 Valid
#3 Invalid
#4 Valid
#5 Invalid
#6 Valid

truth table showing valid and invalid arguments

Premises:

P-> q : if my computer crashes, I will lose all my photos.
~ q : I haven't lost all my photos.

Conclusion:
~p : My computer hasn't crashed.

Argument:

[(p -> q) ∧ ~q] -> ~P

P	q	~P	~q	p -> q	[(p -> q) ∧ ~q]	[(p -> q) ∧ ~q] -> ~P
T	T	F	F	T	F	T

T	**F**	**F**	**T**	**F**	**F**	**T**
F	**T**	**T**	**F**	**T**	**F**	**T**
F	**F**	**T**	**T**	**T**	**T**	**T**

Q.4 a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

Answer (a): **Concept of Union**

In mathematics, the union (denoted by ∪) of a collection of sets is the set of all elements in the collection. It is one of the fundamental operations through which sets can be combined and related to each other.

UNION OF TWO SETS:-

The union of two sets A and B is the set of elements which are in A, in B, or in both A and B. In symbols,

{\displaystyle A\cup B=\{x:x\in A{\text{ or }}x\in B\}}

For example, if A = {1, 3, 5, 7} and B = {1, 2, 4, 6, 7} then A ∪ B = {1, 2, 3, 4, 5, 6, 7}. A more elaborate example (involving two infinite sets) is:

A = {x is an even integer larger than 1}

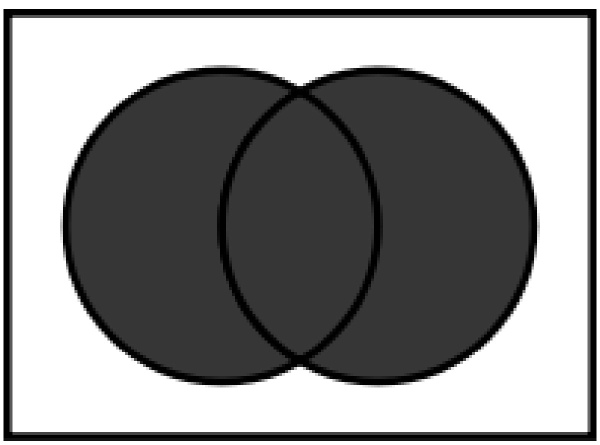
B = {x is an odd integer larger than 1}

{\displaystyle A\cup B=\{2,3,4,5,6,\dots \}}

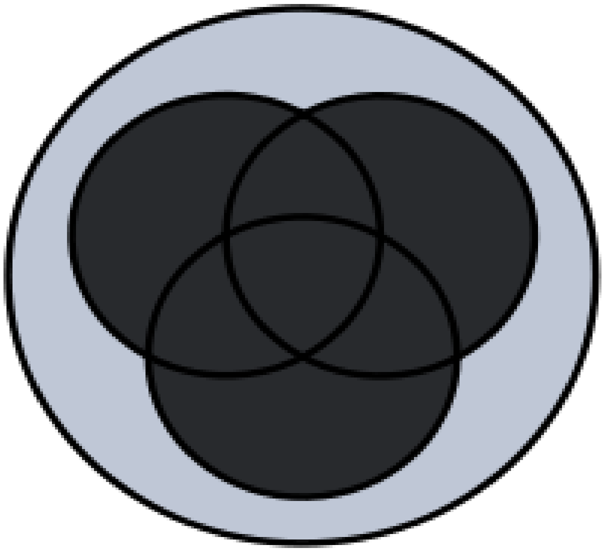
As another example, the number 9 is not contained in the union of the set of prime numbers {2, 3, 5, 7, 11, ...} and the set of even numbers {2, 4, 6, 8, 10, ...}, because 9 is neither prime nor even.

Sets cannot have duplicate elements, so the union of the sets {1, 2, 3} and {2, 3, 4} is {1, 2, 3, 4}. Multiple occurrences of identical elements have no effect on the cardinality of a set or its contents.

Following is the diagram of union .



The above diagram shows union of two(2) sets. i.e A ∪ B
{\displaystyle ~A\cup B}



The above diagram shows union of two(3) sets. i.e A ∪ B ∪ C

MEMBERSHIP TABLE FOR UNION:-

The Membership table for the union of sets A and B is given below.

The truth table for disjunction of two statements P and Q is given below.

In the membership table of Union replace, 1 by T and 0 by F then the table is same as of disjunction.

So membership table for Union is similar to the truth table for disjunction (∨).

FIRST EXAMPLE:

A	B	A ∪ B
1	1	1

1	0	1
0	1	1
0	0	0

TRUTH TABE

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

SECOND EXAMPLE:

C	D	C ∪ D
1	1	1
1	1	1
1	0	1
1	0	1

0	1	1
0	1	1
0	0	0
0	0	0

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b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table. (Note: Examples and truth table should not belongs to your book or slides)

ANSWER (b): **INTERSECTION**

In mathematics, the intersection of two sets A and B, denoted by A ∩ B, is the set containing all elements of A that also belong to B (or equivalently, all elements of B that also belong to A).

INTERSECTION OF TWO SETS OR MORE SETS:-

The intersection of two sets A and B, denoted by A ∩ B, is the set of all objects that are members of both the sets A and B. In symbols,{\displaystyle A\cap B=\{x:x\in A{\text{ and

}}x\in B\}.}

That is, x is an element of the intersection A ∩ B if and only if x is both an element of A and an element of B.

For example:

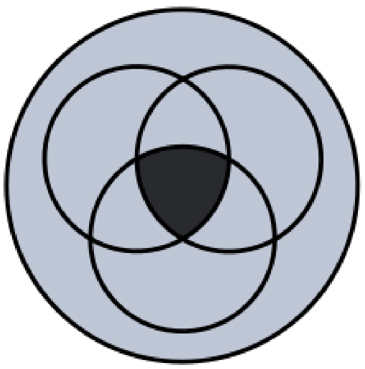
The intersection of the sets {1, 2, 3} and {2, 3, 4} is {2, 3}.

The number 9 is not in the intersection of the set of prime numbers {2, 3, 5, 7, 11, ...} and the set of odd numbers {1, 3, 5, 7, 9, 11, ...}, because 9 is not prime.

Intersection is an associative operation; that is, for any sets A, B, and C, one has $A \cap (B \cap C) = (A \cap B) \cap C$. Intersection is also commutative; for any A and B, one has $A \cap B = B \cap A$. It thus makes sense to talk about intersections of multiple sets. The intersection of A, B, C, and D, for example, is unambiguously written $A \cap B \cap C \cap D$.

Inside a universe U one may define the complement Ac of A to be the set of all elements of U not in A. Now the intersection of A and B may be written as the complement of the union of their complements, derived easily from De Morgan's laws:
$A \cap B = (Ac \cup Bc)c$

Following diagram shows the intersection of three sets.i.e $A \cap B \cap C$



MEMBERSHIP TABLE FOR INTERSCTION:-

The Membership table for intersection of sets A and B is given below.

The truth table for conjunction of two statements P and Q is given below

In the membership table of Intersection, replace 1 by T and 0 by F then the table is same as of conjunction.

So membership table for Intersection is similar to the truth table for conjunction (∧).

FIRST EXAMPLE:

A	B	A ∩ B
1	1	1
1	0	0
0	1	0
0	0	0

TRUTH TABE

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

SECOND EXAMPLE:-

C	D	C ∩ D

1	1	1
1	1	1
1	0	0
1	0	0
0	1	0
0	1	0
0	0	0
0	0	0

Q.5 Explain the concept of Venn diagram with examples.

Answer (a): Venn Diagram

A Venn diagram is a diagrammatic representation of ALL the possible relationships between different sets of a finite number of elements. Venn diagrams were conceived around 1880 by John Venn, an English logician, and philosopher. They are extensively used to teach Set Theory. A Venn diagram is also known as a Primary diagram, Set diagram or Logic diagram

OR

A Venn diagram is an illustration that uses circles to show the relationships among things or finite groups of things. Circles that overlap have a commonality while circles that do not overlap do not share those traits.

Venn diagrams help to visually represent the similarities and differences between two concepts. They have long been recognized for their usefulness as educational tools. Since the mid-20th century, Venn diagrams have been used as part of the introductory logic curriculum and in elementary-level educational plans around the world.

UNDERSTANDING THE VENN DIAGRAM

The English logician John Venn popularized the diagram in the 1880s. He called them Eulerian circles after the Swiss mathematician Leonard Euler, who created similar diagrams in the 1700s.

The term Venn diagram did not appear until 1918 when Clarence Lewis, an American academic philosopher and the eventual founder of conceptual pragmatism, referred to the circular depiction as the Venn diagram in his book A Survey of Symbolic Logic.

APPLICATIONS FOR VENN DIAGRAM

Venn diagrams are used to depict how items relate to each other against an overall backdrop, universe, data set, or environment. A Venn diagram could be used, for example, to compare two companies within the same industry by illustrating the products both companies offer (where circles overlap) and the products that are exclusive to each company (outer circles).

Venn diagrams are, at a basic level, simple pictorial representations of the relationship that exists between two sets of things. However, they can be much more complex. Still, the streamlined purpose of the Venn diagram to illustrate concepts and groups has led to their popularized use in many fields, including statistics, linguistics, logic, education, computer science, and business.

De Morgan’s Laws:

For any sets A and B, then

$$(A \cup B)' = A' \cap B'$$

The compliment of the union is the intersection of each compliment.

$$(A \cap B)' = A' \cup B'$$

The compliment of the intersection is the union of each compliment.

Solving the Cardinal Number Problem:

Define a set for each category in the universal set.

Draw a Venn diagram with as many overlapping circles as the number of sets you have defined.

Write down all the given cardinal numbers corresponding to various given sets.

Starting with the innermost overlap, fill in each region of the Venn diagram with its cardinal number.

Example 1:

A survey of 300 workers yielded the following information: 231 belonged to the Teamsters Union, and 195 were Democrats. If 172 of the Teamsters were Democrats, how many workers were in the following situations?

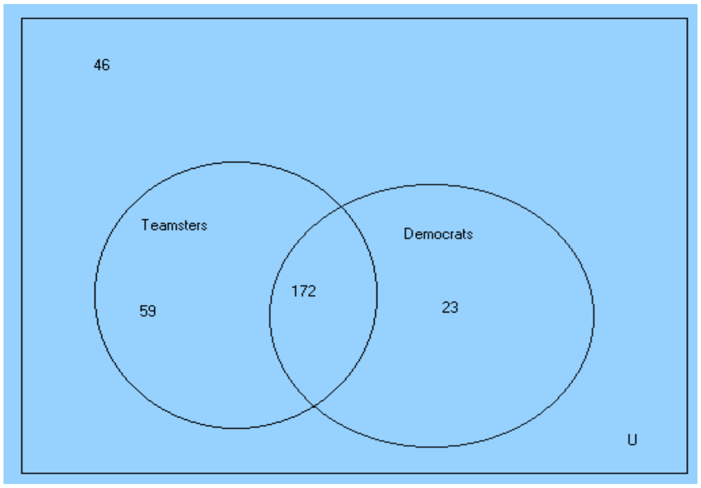
Belonged to the Teamsters or were Democrats

Belonged to the Teamsters but were not Democrats

Were Democrats but did not belong to the Teamsters

Neither belonged to the Teamsters nor were Democrats.

Venn Diagram for example 1:



Explanation:

Let us define T = The event that a worker belongs to the teamsters, and D = the event that a worker is a Democrat. Note that n(U) = 300.

We were given the fact that 172 workers were both in the Teamsters and a Democrat, that is $n(T \cap D)=172$

We are given that n(T)=231 and n(D)=195.

Thus $n(T \cup D) = n(T) + n(D) – n(T \cap D)$

$n(T \cup D) = 231 +195 – 172 = 254$.

Those who are only Democrats and do not belong to the Teamsters are $n(D) – n(T \cap D) = 195 – 172 = 23$.

Those who are only Teamsters but not Democrats are $n(T) – n(T \cap D) = 231 – 172 = 59$.

Those who are neither Teamster or Democrat fall outside of the circles. Use the complimentary law $n(T \cup D) + n((T \cup D)') = n(U)$.

n((T U D)’) = n(U) – n(T U D) = 300 – 254 =46.

b) Given the set P is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set P and indicate all the elements of set P in the Venn diagram.

c) Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

e) Given the set Q = {x : 2x – 3 < 11, x is a positive integer }. Draw and label a Venn diagram to represent the set Q.

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b)

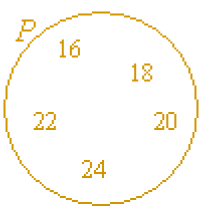
Given the set P is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set P and indicate all the elements of set P in the Venn diagram.

Solution:

List out the elements of *P*.

P = {16, 18, 20, 22, 24} ← ‘between’ does not include 15 and 25

Draw a circle or oval. Label it P . Put the elements in P.



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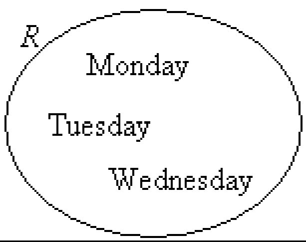
c)

Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

Solution:

Draw a circle or oval. Label it R . Put the elements in R.



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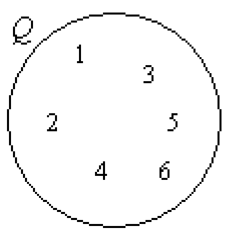
d)

Given the set $Q = \{x : 2x - 3 < 11,\ x \text{ is a positive integer}\}$. Draw and label a Venn diagram to represent the set Q.

Solution:

Since an equation is given, we need to first solve for x.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$



So, $Q = \{1, 2, 3, 4, 5, 6\}$

Draw a circle or oval. Label it Q .

Put the elements in Q.