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SECTION :- A

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Subject Diffrential equation

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Question # 01

Solve the following objective type Questions.

i) The order of matrix A is $m \times p$ and the order of B is $p \times n$ then the order of matrix AB is?

Answer:-

If the order of matrix "A" is $m \times p$ and the order of B is $p \times n$ so the order of matrix AB is "$m \times n$".

ii) The number of non-zero rows in an Echelon form?

Answer:-

The number of non-zero rows in an echelon form is called Rank:

And the number of non-zero rows in echelon form is "One".

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = ?$

Solution.

B is a singular matrix so

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$|B| = (a\times1) - (2\times4) = 0$$

$$|B| = a - 8 = 0$$

As

$$|B| = 0$$

$$0 = a - 8 \quad \boxed{a = 8} \text{ Ans}$$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Solution :-

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

then

$$|A| = 2i(-i) - i(i)$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$\boxed{|A| = 3} \text{ Ans}$$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Solution:-

The given matrix A is a scalar matrix because the diagonal elements are same and non-diagonal are zero.

(Scalar matrix)

vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Solution:-

$$\frac{dy}{dx} + 2xy = y$$

$$\Rightarrow \frac{dy}{dx} = y - 2xy$$

$$\Rightarrow \frac{dy}{dx} = y(1-2x)$$

$$\Rightarrow \frac{dy}{y} = (1-2x)dx$$

$$\Rightarrow \int \frac{1}{y} dy = \int 1 dx - \int 2x dx$$

$$\Rightarrow \ln y = x - \frac{2x^2}{2} + C$$

$$\Rightarrow \ln y = x - x^2 + C$$

Ans

vii) The order and degree of differential equation

<u>Solution:</u>

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

Order 1 , degree = 3

viii) The order and degree of differential equation

<u>Solution:</u>

$$\left(\frac{d^2y}{dx^2}\right)^{B} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$

Order = 2 , degree = 1

ix) The differential equation $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0) = 5$ is

<u>Solution:</u>

Q1(ix)

Solution:

$$2\frac{dy}{dx}+x^2y=2x+3$$

$$2\,dy+x^2y=(2x+3)\,dx$$

$$2dy=(2x+3-x^2y)\,dx$$

$$\int 2dy=\int(2x+3-x^2y)\,dx$$

$$2y=\frac{2x^2}{2}+3x-\frac{yx^3}{3}+C$$

$$2y=x^2+3x-\frac{x^3}{3}y+C$$

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+C \quad \text{—(1)}$$

Put $x=0,\ y=5$

$$5=0+0-0+C$$

$$5=C$$

Or $C=5$

$$y=\frac{x^2}{2}+\frac{3x}{2}-\frac{x^3y}{6}+5$$

Q1)
X.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Solution :

Expand by C1

$$= 1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 - cb^2) - 1(ac^2 - a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$

Ans

QUESTION #02.

i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factor which is linear in a, b, c.

<u>SOLUTION:-</u>

= Expand by R_1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^2(c-b) - ba^2c^2(c-a) + ca^2b^2(b-a)$$

Taking abc common

$$= abc\big(bc(\cancel{a}c-b) - ac(c-a) + ab(b-a)\big)$$

Ans

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ii) Find the Eigen value.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

SOLUTION:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

equation $= |A - \lambda I| = 0$ — (A)

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

Now take determinant.

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand By R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & 1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \text{ ——— } (B)$$

Now Again we take determinant we get

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} \quad \text{expand by } R_1.$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[\left((3-\lambda)(2-\lambda)-(-1)(-1)\right)+1\left((-1)(2-\lambda) - (-1)(-1)\right) -1\left((-1)(-1)-(-1)(3-\lambda)\right)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2+5\lambda+5)+(-3+\lambda)-4(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \rightarrow (1)$$

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$$\Rightarrow + \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1 (6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2 + \lambda - 1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \quad - \text{ (2)}$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$\Rightarrow -1 \left[-(-2 + \lambda - 1) + 1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) \right)$$

$$= -(3 - \lambda) + \lambda^2 - 5\lambda + 5)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\lambda^2 + 6\lambda - 8} \quad - \text{ (3)}$$

put eq (1), (2), (3) in eq (B)

$$= (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda$$
$$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$$
$$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By synthatic division we get

than:-

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \quad \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization method.

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$(\lambda-4)(\lambda-4)$

$\lambda=4,\ \lambda=4$

$\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4$

~~Ans~~ Ans

QUESTION # 03

The rate of change in the form of differential equation is given by. $(x^2+3y^2)dx - 2xy = 0$ Find the general solution at $x=2$ and $y=6$.

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy.$$

Divide B.S by $2xydx$.

we get than

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}.$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{--- (A)}$$

Let we know That

$$y = vx$$

than $\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (i)

put in eq (A)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

multiplying B.S by "2" we g

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v.$$

$$3x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v.$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

xing B.S by $\frac{dx}{dv}$ we get

$$2u\,dv = \frac{1+v^2}{v}\,du$$

multiply b.s by $\frac{v}{x(1+v^2)}$ than,

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on B.S

$$\frac{1}{2}\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c.$$

$$\frac{1}{2}\ln|1+v^2| = \ln x + \ln c.$$

Take e on B.S.

$$\frac{1}{2}\, e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$\frac{1}{2}\, 1+v^2 = xc$$

Ans

Put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \quad \text{—} (*)$$

Put $x=2, y=6$.

$$4+36 = 8c$$

$$c = \frac{40}{8}$$

$c = 5$ Put in $*$

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking root on B.S

$$y = +x\sqrt{5x-x}, \; y = -x\sqrt{5x-1}$$

Or

$$x = \pm\sqrt{5x-1}$$

An

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