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Semester	2nd
Subject	Linear Algebra
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Exam:	Final Exam.

(Answer No 3) a

The four main ~~ingredents~~ ~~or~~ ingredients are

(i) ~~a~~ A set V of vectors.
(ii) A number field K (usually K = IR)
(iii) A Rule for adding vectors (vector addition)
(iv) a way to multiply vectors by a number to produce a new vector (scalar multiplication)

There are, of course, ten rules that there four ingredients must obey.

(Part A)

This is not a vector space. Notice that distvibutivity of scalar multipilication requires

$2u = (1+1)u = u+u$ for any vector u but

$$2 \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$$

which does not equal ~~This~~ ~~could~~

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$$

This could be repaired by taking

$$k \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

(Part B)

This is a vector space. {\ it Although, the question does not ask you to , it is a useful exercise to verify that all \hyperve[vector space] { ten vector Space rules} are satisfied}

(Answer No 4)

(Part A)

whenever $\det M = ad - bc \neq 0$

$$|M| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= (a \times d) - (b \times c)$$

$$= ad - bc \neq 0$$

$$\text{adj } M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

so

$$M^{-1} = \frac{1}{|M|} \text{adj } M$$

$$= \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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(Part B)

unit determinant bit matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right.$$

these all matrices det are zero.

(Part C)

Bit matrices with Vanishing determinant:

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

these all determinant are zero

(Part D)

id = 16054

$$A = \begin{bmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 0 & 6 \\ 5 & 1 & 4 \end{bmatrix}$$

Now finding determinant of A

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 0 & 6 \\ 5 & 1 & 4 \end{vmatrix}$$

by expanding Row one.

$$= 1\begin{vmatrix} 0 & 6 \\ 1 & 4 \end{vmatrix} - 1\begin{vmatrix} 6 & 6 \\ 5 & 4 \end{vmatrix} + 1\begin{vmatrix} 6 & 0 \\ 5 & 1 \end{vmatrix}$$

$$= 1(0-6) - (24-30) + (6-0)$$

$$= \cancel{-6} \cancel{+6} + 6$$

$$= 6$$

$$\boxed{|A| = 6}$$ Ans.

(Answer No 1)

ID = 16054

$$v_1 = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 6 \\ 0 \\ 5 \end{bmatrix},$$

$$v_3 = \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix}$$

by using this

$a_1v_1 + a_2v_2 + a_3v_3 = 0$

for the numbers α_1, α_2 and α_3 not all vanishing.

To find the solution we need to set up a linear system.

$\alpha_1 + 6\alpha_2 + 0 = 0$
$6\alpha_1 + 0 + 5\alpha_3 = 0$
$0 + 5\alpha_2 + 4\alpha_3 = 0$

This can be easily handled using an augmented matrix whose colums are just the vectors we started with

$$\left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 6 & 0 & 5 & 0 \\ 0 & 5 & 4 & 0 \end{array}\right]$$

$R_2 - 6R_1$

$$\left[\begin{array}{ccc|c} 1 & 6 & 0 & 0 \\ 0 & -36 & 5 & 0 \\ 0 & 5 & 4 & 0 \end{array}\right]$$

$6R_1 + R_2$

$$\left[\begin{array}{ccc|c} 6 & 0 & 5 & 0 \\ 0 & -36 & 5 & 0 \\ 0 & 5 & 4 & 0 \end{array}\right]$$

$5R_3 - 4R_2$

$$\left[\begin{array}{ccc|c} 6 & 0 & 5 & 0 \\ 0 & -36 & 5 & 0 \\ 0 & 169 & 0 & 0 \end{array}\right]$$

$\frac{R_3}{169}$

$$\left[\begin{array}{ccc|c} 6 & 0 & 5 & 0 \\ 0 & -36 & 5 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$36R_3 + R_2$

$$\left[\begin{array}{ccc|c} 6 & 0 & 5 & 0 \\ 0 & -36 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

thus we have found a relationship among our ~~~~ other vectors

$6\alpha_1 + 5\alpha_3 = 0$

$-36\alpha_2 + 5\alpha_3 = 0$

$\boxed{\alpha_3 = \frac{-6\alpha_1}{5}}$

$\boxed{\alpha_1 = \frac{-5\alpha_3}{6}}$

$-36\alpha_2 = -5\alpha_3$

$\boxed{\alpha_2 = \frac{5}{36}\alpha_3}$

$$\left(\frac{-5}{6}\alpha_3\right)v_1 + \left(\frac{5}{36}\alpha_3\right)v_2 + \left(\frac{-6}{5}\alpha_1\right)v_3 = 0$$

In fact this is not just one ~~direction~~ relation. but ifinitly many. for any choice of v_2, v_3 the

This elimnates v_2 ~~and~~ v_3 and leaves a pair of linearly independent vectors v_1 and v_3.

(Answer No 2)

(Part 1)

Let

C the total

cost.

~~Factor~~

$$\text{Total material cost} \overset{=T_1}{=} x + y$$
$$= 450 + 400$$
$$= 850$$

$$\text{Total Labour Cost} \overset{=T_2}{=} 250 + 350$$
$$= 600$$

$$\text{Total overhead cost} \overset{=T_3}{=} 150 + 150$$

$$= 300$$

Now find total Cost

$$\text{total Cost} = T_1 + T_2 + T_3$$

$$= 850 + 600 + 300$$
$$= 1750.$$

{Part B}

- $T(u+v) = T(u) + T(v)$

  For all u, v is the domain of T;

- $T(cu) = cT(u)$

  For all u and all scalars c.