NAME: AMIT SINGH

ID: 15478

SUBJECT: DIFFERENTIAL EQUATIONS

SEMESTER: 3RD

PROGRAMME: BS (SOFTWARE ENGINEERING)

Date: ....

Q2) Define 2nd order linear homoge---- -?

Homogenous differential :-

The one which is all the terms involving the unknown functions are collected together on side of equati on the other side is 0

E.g:-

$y'' - 2y' + y = 0$ is homogenous — (i)

$y'' - 2y' + y = x$ (ii) is not homogenous

Non homogenous :-

The non-homogenous differential equations are type form of

$$y'' + Py + qy = f(x)$$

where p & q are constant for each equation we can relate homogenous and complementry equation

APM

Date: ....

Example:

① $y'' + y = \sin 2x$

② $y'' + y' - 6y = \cancel{8}6x$

b) $4y'' - 6y' + 7y = 0$

Sol:-

A second order linear homogenous ODE has form of $ay'' + by' + y = 0$

for equation $ay'' + by' + 4y = 0,$

assume e^{rt}

$y = e^{rt}$

$4((e^{rt}))'' - 6((e^{rt}))' + 7e^{rt} = 0$

$= e^{rt}(4y^2 - 6y + 7) = 0$

$e^{rt}(4y^2 6y + 7) = 0, r = \frac{3}{4} + i\frac{\sqrt{19}}{4}, r = \frac{3}{4} - \frac{\sqrt{19}}{4}$

APM

Date: ....

$$r = \frac{3}{4} + i\frac{\sqrt{19}}{4}, \quad r = \frac{3}{4} - i\frac{\sqrt{19}}{4}$$

for two complex root

$$e^{3/4} + \left(c_1 \cos \frac{\sqrt{19}}{4} 6\right) + c_2 \sin\left(\frac{\sqrt{19}}{7}\right)$$

$$y = e^{3t/4}\left(c_1 \cos \frac{\sqrt{19}t}{4}\right) + c_2 \sin\left(\frac{\sqrt{19}t}{4}\right)$$

ii) $y'' - 4y' - 12y = 3e^{5t}(5x)$

Sol:-

$$y'' - 4y' - 12y = 0$$

Charateristics roots :

$$r^2 - 4r - 12 = (r-6)(r+2) = 0$$

$$\Rightarrow r_1 = 2, \quad r_2 = 6.$$

$$y_c(t) = c_1 e^{-2t} + c_2 e^{6t}$$

$$y_p(t) = Ae^{5t}$$

$$25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae)$$
$$= 3e^{5t} - 7Ae^{5t} - 8e^{5t}$$

APM

Date:

$-7A = 3, \quad A = \frac{3}{7}$

$Y_p(t) = \frac{-3}{7} e^{5t}$

Q2 i)

The characteristic solution is

$16r^2 - 40r + 25 = (4r-5)^2 = 0, r = , 2 = \frac{5}{4}$

The general solution is

$16r^2 - 40r + 25 = (4r-5)^2 = 0, r_1, 2 = \frac{5}{4}$

General Derivation:

$y(t) = c_1 e^{5t/4} + C_2 t e^{5t/4}$

$y'(6) = \frac{5}{4} c_1 e^{5t/4} + C_2 e^{5t/4} + C^{5t/4}$

$3 = y(0) = c_1$

$\frac{-9}{4} = y'(0) = \frac{5}{4} c_1 + c_2$

$c_1 = 3, \quad c_2 = -6$

$y(t) = 3e^{5t/4} - 6te^{5t/4}$ Ans

APM

Scanned with CamScanner

Date: ....

Q2 ii) $y'' + 14y' + 49y = 0$, ----

Sol:-

$r^2 + 14r + 49 = (r+7)^2 = 0, r_1, r_2 = 7$

$y(6) = c_1 e^{-7t} + c_2 t e^{-7t}$

$y(6') = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 e^{-7t}$

$-1 = y(-4)\ c_1 e^{28} - 4c_2 e^{28}$

$5 = y'(-4) = 7c_1 e^{28} + 28 c_2 e^{28} =$

$-7c_1 e^{28} + 29 c_2 e^{28}$

$c_1 = -9e^{-28}$ Ans

iii) $y'' - 4y' + 9y = 0$, -----

Sol:-

$r^2 - 4 + 9 = (r-2)^2 = 0, r_1, r_2 = 2$

$y(6) = c_1 e^{2t} + c_2 t e^{2t}$

$y'(6) = 2c_1 e^{5t} + c_2 e^{5t} + 2c_2 t e^{2t}$

$12 - y(0) = c_1$

$-3 = y'(0) = 2c_1 + c_2$

$c_1 = 12, \quad c_2 = -27$

$y(t) = 12e^{t} - 27te^{2t}$ Ans

APM

Date: ....

(iv) $y'' - 8y' + 17y = 0$ - - - -

$r_1 = r_2 = 4$

$y(x) = c_1 e^{4x} + c_2 e^{4x} x$

$y(t) = c_1 e^{4t} + c_2 t e^{4t}$, $\frac{dt}{dt}\left[te^{4t}\right]$

$= 4te^{4t} + e^{4t}$

$y'(t) = 4c_1 e^{4t} + 4c_2 te^{4t} + c_2 e^{4t}$

$-2 = c_1 e^{4(0)} = c_2(0) e^{4(0)}$

$-2 = c_1, c_1 = -2$

$-\frac{22}{3} + 4e \; c_1 e^{4(0)} + 4c_1(0)e^{4(0)}$
$+ c_2 e^{4(0)}$

$-\frac{22}{3} = 4(-2) + c_2 = c_2 = 1/3$

$y(x) = \frac{1}{3} e^{4x} (x - 6)$ Ans

APM

Date: ....

Q3 Define Laplace form ---?

LAPLACE FORM:

It's a technique for solving differetial equation here d.e of first domain is tranformed in algebric equation of frequency domain form

$$\mathcal{L}\left[g(t)\right) = 1$$

$$\mathcal{L}\left[4(t)\right) = \frac{1}{s}$$

$$F(s) = \int_0^1 F(t) \cdot e^{-st} \, dt$$

e.g: $\int_0^8 e^{at} \cdot dt$

$\mathcal{L}\{(\sin at)\}$

APM

Date: ....

i) $f(t) = 6(e^{-5t}) + e^{3t} + 5(t^3) - 9$

Sol:-

$$F(S) = \frac{6\,1}{S-(-5)} + \frac{1}{S-3} + \frac{5\,3!}{S^{3+1}} - 9\frac{1}{S}$$

$$= \frac{6}{S+5} + \frac{1}{S-3} + \frac{30}{S^4} - \frac{9}{S}$$

ii) $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

Sol:-

$$G(S) = \frac{4.S}{S^2+(4)^2} - \frac{9\,4}{S^2+(4)^2} + \frac{2\,S}{S^2+(10)}$$

$$= \frac{4S}{S^2+16} - \frac{36}{S^2+16} + \frac{2S}{S^2+10}$$

Q3) $h(t) = e^{3t} + \cos(3t) \cdot e^{\wedge 3t}$ (r) St

$$H(S) = \frac{3\,2}{S^2-(2)^2} + \frac{3\,(2)}{S^2+(2)^2}$$

$$= \frac{6}{2^2-4} + \frac{6}{S^2+4}$$

APM

Date: ....

Q4)i $y'' - 10y' + 9y = 5t$ ----

Sol:-

First convert into D.E

$$\mathcal{L}\{y''\} - 10\,\mathcal{L}\{y''\} + 9\,\mathcal{L}\{y''\} = \mathcal{L}\{5t\}$$

$$s^2 y(s) - 8y(0) - y'(0) - 10(s-1(s) - y(10)) + 9y(3) = \frac{5}{s^2}$$

$$(8^2 - 10s + 9)\, Y(8) + s - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)(s-1)} + \frac{12-s}{(s-9)(0-1)}$$

Combining terms

$$Y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$5 + 12s^2 - s^3 = As(s-9)(s-1) + B(s-9)(s-1) + C^2 s(s-1) + Ds^2(s-9)$$

APM

Date: ....

$S = 0 \quad\quad 5 = 9B$

$S = 1 \quad\quad 16 = -8D$

$S = 9 \quad\quad 248 = 648C$

$S = 2 \quad\quad 45 = -14A + \frac{\cancel{43}\ \cancel{45}}{81}$

$B = \frac{5}{9} = \frac{50}{81/s} + \frac{3/9}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$

$y(6) = \frac{5D}{81} + \frac{5t}{9} + \frac{3/\cancel{8}e^{9t}}{81} - 2e^t$

Ans

ii) $y'' - 6y' + 15y = 2\sin(3t), y(0) \cdots$

Sol:-

$S^2 x(s) - Sy(0) = -6(Sx(S) - y(0)) +$
$15y(S) = 2\frac{3}{S^2+9}$

$(S^2 - 6s + 15)Y(s) + S - 2 = \frac{6}{S^2+9}$

$Y(S) = \frac{-S^3 + 2S^2 - 9S + 24}{(S^2+9)(S^2-6s+15)}$

APM

Date: ....

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2-6s+15}$$

$$-s^3 + 2s^2 - 9s + 24 = (As+D)(s^2-6s+15) + (5s+0)(s^2+9)$$

$$= (A+C)s^3 + -(6A+B+D)s^2 + (15A-6B + 9C) + 15B + 9D$$

s^3 : $A + C = -4$
s^2 : $-(A+B+D = 2$
s^1 : $15A - 6B + 9C = -9$
s^0 : $15B + 9D = 24$

$A = 1/10$
$B = -11/10$
$C = -1/10$
$D = 5/2$

$$Ys = 1/10 \left(\frac{s+1}{s^2+9} + \frac{-11s + 25}{s^2-6s+15} \right)$$

$$= \frac{1}{10} \left(\frac{s+1}{s+9} + \frac{-11(6-3+3)+25}{(s-3)^2+6} \right)$$

$$= \frac{1}{10} \left(\frac{s}{s^2+9} + \frac{13/3}{s^2+9} \frac{-11(s-3)}{(s-3)^2+6} \right)$$

$$y(6) = \frac{1}{10} \left(\cos 3t + \frac{1}{3} \sin 3t - 11e^{3t} \cos(\sqrt{6}t) - \frac{8}{\sqrt{6}} (3^t \sin(\sqrt{6}t) \right)$$

APM