

Iqra National University
Department of electrical
Engineering

Assignment

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Course Details

Course Title: Calculus Module 03

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Program BEE

(Page 1)

Q1: part A: $\lim_{h \rightarrow 0} 2+h - 2h$

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Sol: $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0} = \frac{0}{0}$$

So

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Multiply and dividing $\sqrt{2+h} + \sqrt{2}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h} - \sqrt{2})(\sqrt{2+h} + \sqrt{2})}{(h)(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{2+h - 2}{h(\sqrt{2+h} + \sqrt{2})}$$

P. 1.0

< Page 2 >

$$= \lim_{h \rightarrow 0} \frac{K}{K(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

putting limit

$$= \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

part B:

$$y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

Sol: $y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$

$$= \frac{dy}{dx} = \frac{d}{dx} \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

$$= \left(x + x^{-1}\right) \frac{d}{dx} \left(x - x^{-1} + 1\right) + \left(x - x^{-1} + 1\right) \frac{d}{dx} \left(x + x^{-1}\right)$$

$$= \left(x + x^{-1}\right) \left(1 + x^{-2}\right) + \left(x - x^{-1} + 1\right) \left(1 - x^{-2}\right)$$

< Page 3 >

$$= \left(x + \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right) \left(1 - \frac{1}{x^2}\right)$$

$$= x + x \frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} + x - x \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x^3} + 1$$

$$= 2x + 2 - \frac{1}{x^2} + \frac{1}{x^3} \quad \text{Ans}$$

Q2 (a)

$$S = 160t - 16t^2 \text{ ft}$$

So Given $S = 160t - 16t^2 \text{ ft}$

a) velocity $v = \frac{ds}{dt} = \frac{d}{dt} (160t - 16t^2)$

$$= \frac{d}{dt} 160t - \frac{d}{dt} 16t^2$$

$$v = 160 - 32t$$

Maximum
Height

$$v = 0$$

So $160 - 32t = 0$

$$\frac{160}{32} = \frac{32t}{32}$$

$$t = 5 \text{ seconds.}$$

p.T.O.

< Page 4 >

$$S_{\max} = S(5) = 160(5) - 16(5)^2$$

$$S_{\max} = 400\text{ft}$$

b) Given that $S = 256\text{ft}$

Then $S = 256\text{ft}$

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$\frac{16}{16} (t^2 - 10t + 16) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 0$$

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t-8) - 2(t-8) = 0$$

$$(t-8)(t-2) = 0$$

$$t-8 = 0 \quad t-2 = 0$$

$$t = 8\text{sec} \quad t = 2\text{sec}$$

Since $v = 160 - 32t$
 $t = 2\text{s}$

$$v(2) = 160 - 32(2) \\ = 160 - 64$$

$$v(2) = 96\text{m/s} \Rightarrow \text{velocity upwards}$$

$$t = 8\text{s}$$

P.T.O

< Page 5 >

$$v(t) = 160 - 32(8) \\ = 160 - 256$$

$$= -96 \text{ m/s} \Rightarrow \text{velocity } \downarrow$$

Since $v = 160 - 32t$

Time (t) given = 5 sec

$$\text{So } v = 160 - 32(5) \\ v = 160 - 160 \\ v = 0$$

So to find acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt} 0$$

$$a = 0$$

$$a = 0 \text{ m/s}^2 \text{ Ans}$$

Q3 (a) (i) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where

Sol
7

$$y = x^4 - 2x^2 + 2$$

$$y = x^4 - 2x^2 + 2$$

$$\frac{dy}{dx} = \frac{d}{dx} (x^4 - 2x^2 + 2)$$

$$= \frac{d}{dx} x^4 - \frac{d}{dx} 2x^2 + \frac{d}{dx} 2$$

$$= 4x^3 \frac{dx}{dx} - 2 \times 2x \frac{dx}{dx} + 0$$

$$= 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the tangent is horizontal.

then $\frac{dy}{dx} = 0$

< Page 7 >

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad x^2 - 1 = 0$$

$$\frac{4x}{4} = \frac{0}{4}$$

$$x^2 - 1 = 0$$

$$x = 0$$

$$\sqrt{\frac{x^2}{x^2}} = \frac{1}{\sqrt{1}}$$

$$x = 0$$

$$x = \pm 1$$

So $x = 0, 1, -1$

Their corresponding point in

$$y = x^4 - 2x^2 + 2$$

For

$$y = x^4 - 2x^2 + 2$$

$$= (0)^4 - 2(0)^2 + 2$$

$$= 0 - 0 + 2$$

$$\checkmark y = 2$$

$$x = 1$$

$$y = x^4 - 2x^2 + 2$$
$$= (1)^4 - 2(1)^2 + 2$$

$$= 1 - 2 + 2$$

$$y = 1$$

< page 8 >

$$x = -1$$

$$y = x^4 - 2x^2 + 2$$

$$= (-1)^4 - 2(-1)^2 + 2$$

$$= 1 - 2 + 2$$

$$= 1$$

hence $(0, 2)$, $(1, 1)$, $(-1, 1)$

Thank you
Sir!