Differential Equation

T. Name: Latif Jan

S. Name: Taimoor Khan

Assignment No: 2

S. ID: 13579

Q 1

(a) = $x' = \sqrt{x}$

$$\frac{dx}{dy} = \sqrt{x}$$

separation of variables.

$$dx = \sqrt{x}\, dy$$

~~$\frac{dx}{dy} = \sqrt{x}$~~

$$\frac{dx}{\sqrt{x}} = dy$$

taking $\int$ on both sides

$$\int (x)^{-1/2}\, dx = \int dy$$

$$\frac{x^{-1/2+1}}{-1/2+1} = y + c$$

$$\frac{x^{1/2}}{1/2} = y + c$$

$$2x^{1/2} = y + c$$

Q 3 (a)

when $x(0) = 1$

$$2 \cdot (1) = y + c$$

$$\boxed{2 = y + c}$$

(b) $x' = e^{-2x}$

$\frac{dx}{dy} = e^{-2x}$

Separation of variables.

$\frac{dx}{e^{-2x}} = dy$

Taking $\int$ on both sides.

$\int \frac{dx}{e^{-2x}} = \int dy$

$\int e^{2x}\, dx = \int dy$ — (i)

let $2x = u$

$\frac{d}{dx}(2x)\, dx = \frac{d}{du}(u)\, du$

$2dx = du$

$dx = \frac{1}{2} du$

Put all these value in eq (i).

$\int e^{u} \cdot \frac{1}{2} du = \int dy$

$$\int \frac{e^u}{2}\, du = y + c \qquad \because \int e^u = e^u$$

$$\frac{e^u}{2}\, du = y + c$$

Replace u by $2x$

$$\frac{e^{2x}}{2} = y + c$$

Q(3) (b)

when $x(0) = 1$

$$\frac{e^{2(1)}}{2} = y + c$$

$$\boxed{\frac{e^2}{2} = y + c}$$

(c) $y' = 1 + y^2$

Sol

$$\frac{dy}{dx} = 1 + y^2$$

separation of variable

$$\frac{dy}{1+y^2} = dx$$

$$\int \frac{dy}{1+y^2} = \int dx$$

so standard integral of

$$\left\{ \int \frac{dy}{1+y^2} = \tan^{-1} y \right.$$

$$\boxed{\tan^{-1} y = x + C}.$$

Q4

(a) $x' = \frac{2x}{t+1}$

Sol

$\frac{dx}{dt} = \frac{2x}{t+1}$

solve by separation of variable

$\frac{dx}{2x} = \frac{1}{t+1} dt$

take "$\int$"

$\int \frac{dx}{2x} = \int \frac{1}{t+1} dt$

$\frac{1}{2} \int \frac{1}{x} dx = \int \frac{1}{t+1} dt$

$\frac{1}{2} \ln|x| = \ln|1+t| + \ln c$

$\ln|x^{1/2}| = \ln\{|1+t| \cdot c\}$

$x^{1/2} = \frac{\ln}{\ln}\{|1+t| \cdot c\}$

$\boxed{x^{1/2} = (1+t)c}$