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Section: C

Subject: Intr to Structural Dynamic and Earthquake Engg

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Date: 29 Jun 2020

Final Paper

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QNO: 1

Ans:

Given data:

- A beam is pulled in downward direction = 1/2 inch
- Ignor the self weight of a beam as well a damping effect.
- E = 2900 ksi
- I = 150 in^4
- δ_{st} = deflection due to 7743 lb static load.

Required data:

- Natural time period of system = ?
- Develop and solve equation of motion for vibration = ?

Solution:

The general E.O.M for S.DOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t) \longrightarrow (i)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence general EOM becomes

$$ku + m\ddot{u} = 0 \longrightarrow (ii)$$

$K = 3EI/L^3$

put value

$$= \frac{3 \times 29000\text{ksi} \times 150\,\text{in}^4}{10 \times 12\,\text{in}}$$

$K = 7.55\ K/in$

To eliminat mistake during calculation, use fundamental units like lb, ft, sec etc

So

$K = 7.55\ K/in = \boxed{90625\ lb/ft}$

$m = \frac{7743\ lb\ sec^2}{32.2\ ft}$

$\boxed{m = 240.465\ slug}$ rad/sec

$\omega_n = \sqrt{K/m} = \sqrt{\frac{96025}{240.46}}$

$\omega_n = 19.983\ rad/sec$

$T_n = 2\pi/\omega_n = 2\pi/19.983 = 0.314\ sec$

substituting the corresponding value in eq(ii)

$90625u + 240.4\ddot{u} = 0$

where "K" is in lb/ft and 'm' is in lb sec/ft²

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = \frac{1}{2''} = \frac{1}{24}\text{ ft} \quad \text{and} \quad \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.9t) + 0 =$$

$$\left(\frac{1}{24}\right) \times \cos(19.9t)$$

Equivalent static force at any time "t" is

$$f_s(t) = K.u(t) = \frac{90625 \times \cos(19.9t)}{24}$$

$$f_s(t) = 7552.08 \times \cos(19.9t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

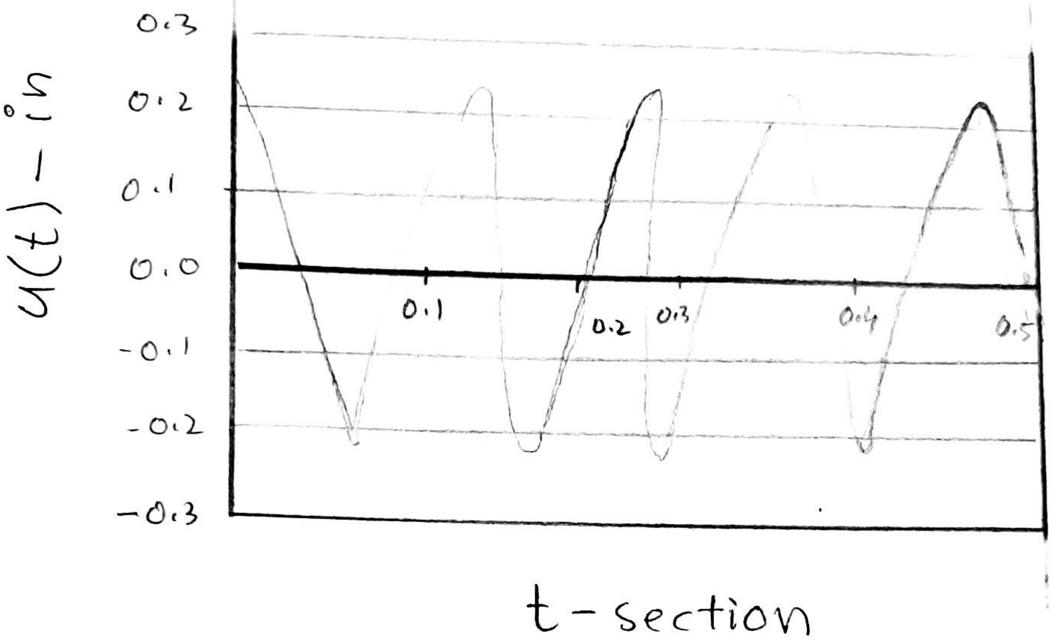
$$= \frac{1}{24}\ ft$$

Amplitude of equivalent static force, f_{s0}

$$Ku_0 = 90625 \times \frac{1}{24}$$

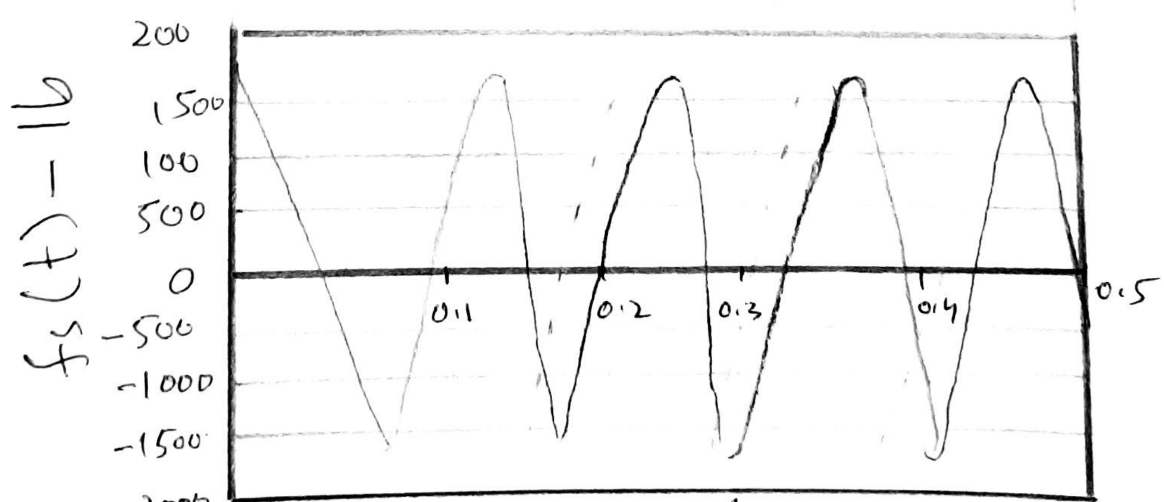
$$\boxed{Ku_0 = 3776.04\ lb}$$

Undamped Free Vibration



Variation of displacement with time.

Undamped Free Vibration



variation of equivalent static Forces with time

Q NO: 2

Given Data:

- Damping ratio of reinforce concrete with considerable cracking = 3-5 %
- So we take $\zeta = 3\%$
- Other data are taken from question No: 1

Required value:

- Develop and solove the eqn of motion for vibration at free end = ?
- Develop an equation showing veriation in equivalent state forces with time = ?

Solution :-

As we know that

EOM (equation of motion) for damped free vibration is;

$$Ku + C\dot{u} + m\ddot{u} = 0 \longrightarrow (1)$$

As we know that from question no. 1 data

$K = 90625$ lb/ft

$m = 240.46$ rad/sec

$W_n = 19.983$

As we know that

$C = \zeta \times 2 m W_n$

$C = 0.03 \times 2 \times 240.46 \times 19.98$

$C = 288.263$ lb sec/ft

P.T.O

By putting value in eq (1) we get

$$90625u + 288.263\dot{u} + 240.4\ddot{u} = 0$$

Solution to the EOM for damped free vibration is :

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\sin\omega_D t\right]$$

Here;

$$\omega_D = 19.983 \text{ rad/sec}$$

$$u(t) = e^{-(0.03\times19.983\times t)}\left[\frac{1}{24}\times\cos(19.98464t) + \frac{1}{19.983}\times\left[0 + \frac{1}{24}\times 0.03\times19.983\times\sin(19.983)\right]\right]$$

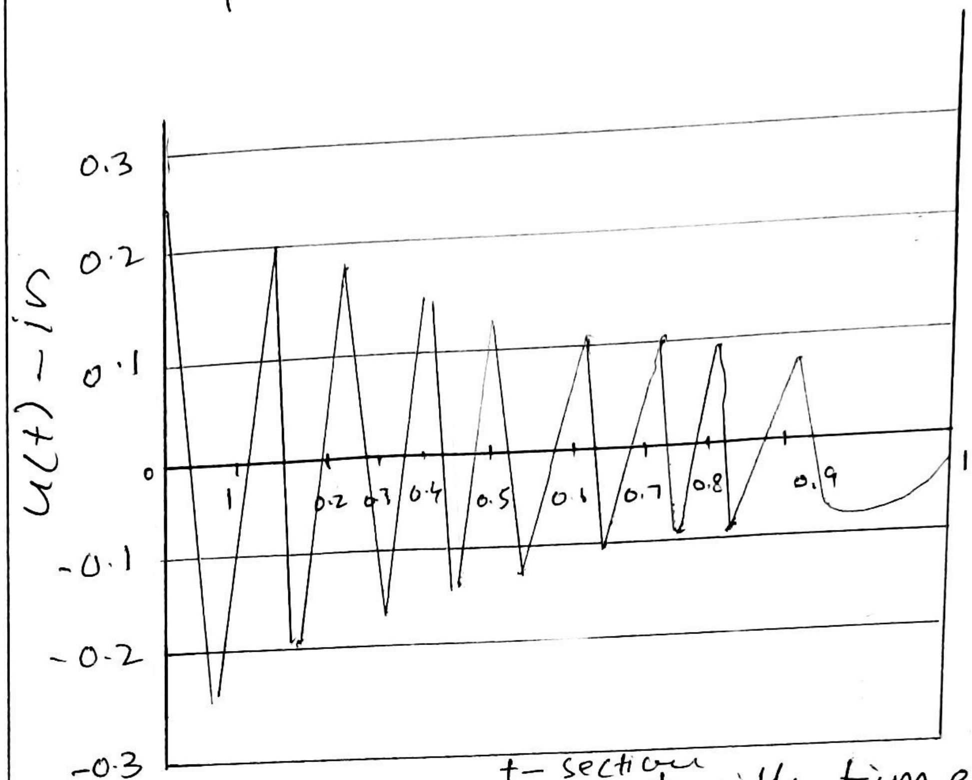
$$u(t) = e^{-(0.599t)}\left[0.0416\times\cos(19.983t) + 0.024\sin(19.983)\right]$$

We know
$f_s(t) = k\cdot u(t)$

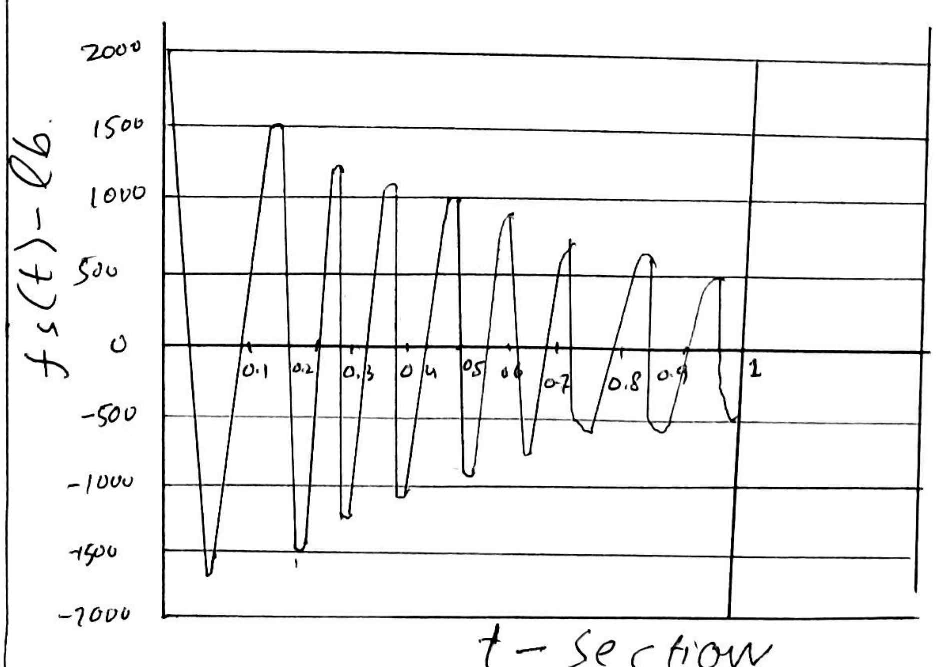
$$f_s(t) = 90625\times u(t)$$

$$f_s(t) = e^{-0.599t}\left[380625\cos(19.983t) + 2175\sin(19.983t)\right]$$

Damped Free Vibration



veriation of displacement with time



.. veriation of equivalent static force with time.

QNO: 3

Given Data:

- Force = 60 kips
- $u_1 = 7743/1000 = 7.743$
- After; $j = 7$ (cycles)
- Completed = 3.57 sec

Required data

(a) Dumping ratio

(b) Natural period of undamped vibration

(c) Stifness of structures

(d) weight of tank

(e) Dumping Coefficient

(f) No. of cycles to reduce the displacement amplitude to 0.5"

Solution:

$u_1 = 7.7''$

after $j = 7$, $u_{j+1} = u_8 = 0.9''$

(a) Damping Ratio $= \zeta = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left(7.7/0.9\right)$$

$$\zeta = 0.0488 = 4.88\%$$

(b) $T_n = ?$

7 cycles of vibration are completed in 3.57 seconds

⇒ Time required to complete one cycle $= 3.57/7$

$$T_n = 0.51 \text{ sec}$$

Now

$$\omega_D = \omega_n \sqrt{(1-\xi^2)}$$

$$2\pi/\omega_D = 2\pi/(\omega_n\sqrt{1-\xi^2})$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\xi^2}}$$

$$T_n = T_D \times \sqrt{1-\xi^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.488)^2}$$

$$\Rightarrow T_n = 0.445 = 0.46 \text{ sec}$$

(c) Stiffness of Structure = K = ?

$$K = \frac{F \cdot \cos\theta}{7.743} = \frac{60\cos 60}{7.743}$$

~~K = [illegible]~~

~~K = 3874.46 lb/ft~~

$$\boxed{K = 46493.60\ lb/f}$$

(d) weight of the tank, w=?

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \times g}{W}}$$

$$\omega_n^2 = K \times g / W$$

$$\Rightarrow \quad W = K \times g / \omega_n^2$$

also

$$\omega_n = 2\pi / T_n$$

$$W = Kg \Big/ \left(\frac{4\pi^2}{T_n^2}\right) = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{46493.60 \frac{lb}{ft} \times 32.2 \frac{ft}{sec^2} (0.46\, sec)^2}{4\pi^2}$$

$$W = 46493.60 \times 32.2 \times \left(\frac{(0.46)^2}{4(3.14)^2}\right)$$

$$W = 8032.40 \text{ lb}$$

$$\boxed{W = 8.032 \text{ K}}$$

(e) Dumping Coefficient :

"c" = ?

We know that

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow \quad c = \zeta(2m\omega_n) = \zeta\left(2m\left(\frac{2\pi}{T_n}\right)\right)$$

put value

$$c = \frac{0.0462\left(2\left(\frac{8032.40}{32.2}\right)\right)}{0.46 \text{ sec}}$$

$$\boxed{c = 50.107 \text{ lb sec/ft}}$$

('f') No of cyclos to reduce the displacement amplitude to 0.5"

$j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\frac{1}{2(3.14)(0.0482)} \ln\left(\frac{7.743}{0.5}\right)$$

$$= 9.1 \quad \boxed{J = 9 \text{ cycles}}$$

OR