

19th national university

Calculus

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Subject calculus

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(1)

$$\text{Q1} \int_0^{\pi/4} (1 - \sin t)^{3/2} \cos 2t \, dt$$

Solution:

$$\int_0^{\pi/4} (1 - \sin t)^{3/2} \cos 2t \, dt$$

Integration by part

$$(1 - \sin t)^{3/2} (\cos 2t) = (1 - \sin t)^{3/2} (\cos 2t) \cdot 1$$

$$\int \cos 2t \, dt$$

$$= \frac{3}{2} \left((1 - \sin t)^{1/2} \cos 2t - \frac{\sin 2t}{2} \right)$$

$$\int (1 - \sin t)^{3/2} (\cos 2t) \int_0^{\pi/4} = \frac{6}{4} \int_0^{\pi/4} (1 - \sin t) \cos 2t \sin t \, dt$$

Now let

$$y = \sin t$$

$$\frac{dy}{dt} = \cos t$$

$$dy = \cos t \, dt$$

$$(1 - \sin t)^{3/2} (\cos 2t) = \frac{6}{4} \int_0^{\pi/4} (1 - y)^{3/2} y \, dy$$

Now let

$$t = 1 - y$$

(2)

$$\frac{dt}{du} = -1$$

$$dt = -du$$

$$(1 - \sin t)^{3/2} (\cos 2t) = \frac{6}{4} \int_0^{\pi/4} (t)^{1/2} (1-t)^2 dt$$

~~$$-\frac{6}{4} \int_0^{\pi/4} (u^{1/2}) (1+u^2-2u) dt$$~~

~~$$-\frac{6}{4} \int (u^{1/2}) (1+u^2-2u) dt$$~~

~~$$-\frac{6}{4} \int_0^{\pi/4} (u^{1/2} + u^{5/2} - 2u^{3/2}) dt$$~~

~~$$-\frac{6}{4} \left[\frac{u^{3/2}}{3/2} + \right]$$~~

$$= -\frac{6}{4} \int_0^{\pi/4} (1-2t+t^2) (t^{1/2}) dt$$

$$= -\frac{6}{4} \int_0^{\pi/4} (t^{1/2} - 2t^{3/2} + t^{5/2}) dt$$

Apply integrat

$$= -\frac{6}{4} \int_0^{\pi/4} \left[\frac{t^{5/2+1}}{5/2+1} - \frac{2t^{3/2+1}}{3/2+1} + \frac{t^{1/2+1}}{1/2+1} \right] dt$$

(3)

$$= -\frac{6}{4} \int_0^{\pi/4} \left[\frac{2t^{2/3}}{7} - \frac{4}{5} t^{5/2} + \frac{2}{3} t^{3/2} \right] dt$$

$$= -\frac{6}{4} \left[\frac{2}{7} (\sin t)^{7/4} \int_0^{\pi/4} - \frac{4}{5} (\sin t)^{5/4} \int_0^{\pi/4} + \frac{2}{3} (\sin t)^{3/4} \int_0^{\pi/4} \right]$$

$$= -\frac{6}{4} \left[\frac{2}{7} (\sin 45^\circ)^{7/4} - \frac{4}{5} (\sin 45^\circ)^{5/4} + \frac{2}{3} (\sin 45^\circ)^{3/4} \right]$$

$$= -\frac{6}{4} \left(\frac{2}{7} (\sin 45^\circ)^{7/4} - \frac{4}{5} (\sin 45^\circ)^{5/4} + \frac{2}{3} (\sin 45^\circ)^{3/4} \right)$$

$$= -\frac{6}{4} (0.28 + 0.085 - 0.33)$$

$$= -\frac{6}{4} (0.035)$$

$$= 0.0525 \text{ Ans}$$

$$\text{Q2} \int_0^1 (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy$$

Solution:

$$\int_0^1 (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) dy$$

By using substitution method

$$\text{Let } x = 4y - y^2 + 4y^3 + 1$$

$$\frac{dx}{dy} = 4 - 2y + 12y^2 \Rightarrow dx = (4 - 2y + 12y^2) dy$$

$$\int_0^1 x^{-2/3} dx$$

$$= \frac{x^{-2/3 + 1}}{-2/3 + 1} + C$$

$$= \frac{x^{1/3} + C}{1/3}$$

$$= 3x^{1/3} + C$$

$$\Rightarrow (3(4y - y^2 + 4y^3 + 1))^{1/3} + C$$

Putting the limits

$$= (3(4(1) - (1)^2 + 4(1)^3 + 1))^{1/3} + C$$

(5)

$$= 3(4 - y + 4 + y)^{1/3} + C$$

$$= 3(8)^{1/3} = C \longrightarrow \text{arbitrary constant}$$

$$= 3(2)^{1/3}$$

$$= 3(2) \Rightarrow 6$$

$$\boxed{= 6}$$



~~Q2~~