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SECTION : "B"

SUBJECT : Earth Quake Engineering

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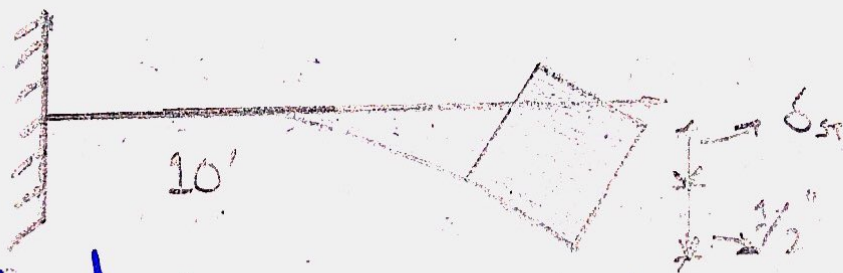
# ANSWER TO QUESTION # 01

## Given data:

$$E = 29,000 \text{ ksi}$$

$$I = 150 \text{ in}^4$$

$\delta_{st}$  = Deflection due to  
7747 lb static  
load beam is pulled  
 $\frac{1}{2}$ " downward.



## Required:

- Natural time period of system.
- Develop  $\zeta$ , solved the equation of motion.
- Draw graph to show the variation of equivalent static forces with time.

## Solution:

General equation of motion for SDOF System is;

$$k_u + C_u + m_u = P(t)$$

Since system is undamped ( $C=0$ )  
under going for vibration  $P(t)=0$

Hence general EOM becomes

$$k_u + m_u = 0 \rightarrow \textcircled{A}$$

$$k = \frac{3EI}{L^3} \Rightarrow \frac{3(29000 \frac{\text{K}}{\text{in}^2})(150 \text{ in}^4)}{(10 \times 12 \text{ in})^3}$$

$$k = 7.55 \text{ K/in}$$

→ In order to simplify & eliminate chances of mistake during calculation. It is more appropriate to use fundamental units like lb, ft, sec or kg, m, sec.

$$k = 7.55 \text{ K/in}$$

$$k = 90625 \text{ lb/ft}$$

$$m = \frac{W}{g} = \frac{7747}{32.2} \Rightarrow 240.59 \text{ slug}$$



$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240 \cdot 89}}$$

$$\omega_n = 19.41 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.41} \Rightarrow 0.324 \text{ sec}$$

Put the value of  $m$  &  $k$   
in eq (A)

$$k_u + m_u = 0$$

$$90625 + 240 \cdot 89 = 0$$

where "k" is in lb/ft and  
"m" is in lb sec<sup>2</sup>/ft<sup>2</sup> = slugs

General solution of EOM for undamped for vibration is

$$U(t) = U(0) \cos(\omega_n t) + \frac{U'(0)}{\omega_n} \sin(\omega_n t)$$

$$U_n(0) = \frac{1}{2} = \frac{1}{24} \text{ ft and } U'(0) = 0$$

$$U(t) = \frac{1}{24} \times \cos(19.39t) + 0$$

$$= \frac{1}{24} \times \cos(19.41t)$$

Equation static force at any time

"t" is  $f_s(t) = k \cdot u(t)$

$$\Rightarrow \frac{90625 \times \cos(19.41t)}{24}$$

$$\Rightarrow 3776 \times \cos(19.41t)$$

Amplitude of dynamics displacement for undamped vibration is

$$u_0 = \sqrt{\left[ (u(0))^2 + \left( \frac{v(0)}{\omega_n} \right)^2 \right]}$$

$$= \sqrt{\left[ \left( \frac{1}{24} \right)^2 + 0 \right]}$$

$$= \sqrt{\left( \frac{1}{24} \right)^2}$$

$$= \frac{1}{24} \text{ ft}$$

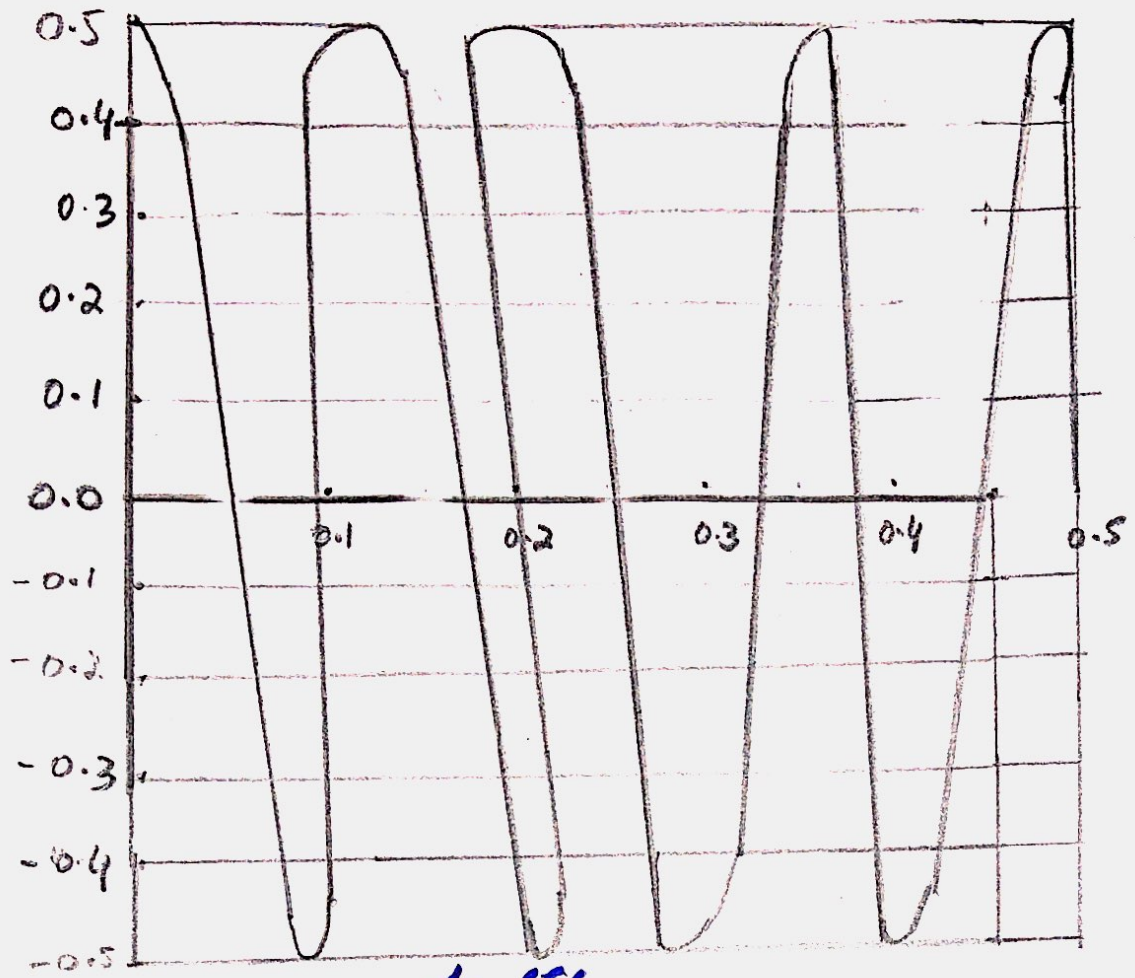
Amplitude of equivalent static forces.

$$k_{eq} = 90625 \times \frac{1}{24}$$

$$k_{eq} = 3776 \text{ lb}$$

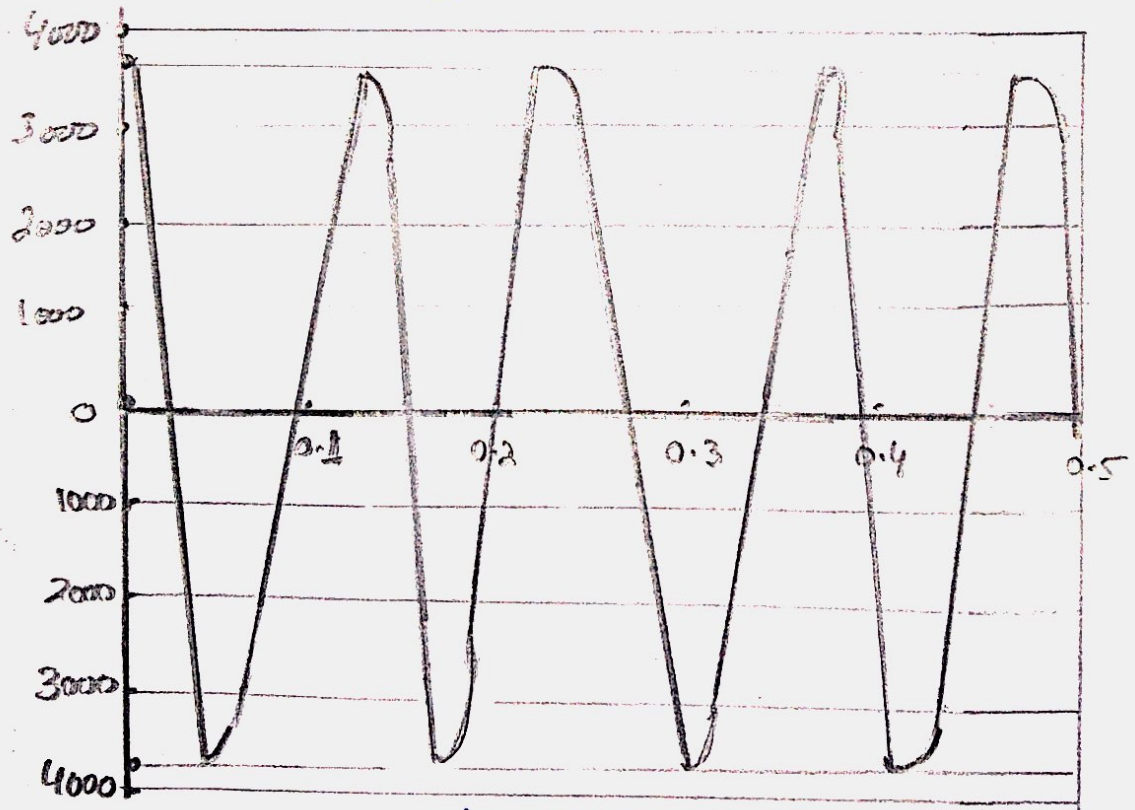


$u(t) - 17$



$t - \text{SEC}$

$F_s(t) - 1b$



$t - \text{Sec}$

# ANSWER TO QUESTION #02

Given data :

(damping ratio) of Reinforced Concrete with consideration of cracking = 3-5%

$$= 3\%$$

Data of Beam from Q=1

Required :

Develop and solve the equation showing variation in equivalent static force with time.

Draw graph to show that variation of displacement with time and the ratio variation of equivalent static force with time.

Solution ::

EOM for damped force vibration is

$$k u + C \dot{u} + m \ddot{u} = 0 \rightarrow (A)$$

As From Question # 1

$$k = 90625 \text{ lb/ft}$$



$$m = 240.59 \text{ lb sec}^2/\text{ft}$$

$$\omega_n = 19.41 \text{ rad/sec}$$

$$c = \zeta \times 2m\omega_n$$

$$c = 3\% \times 2(240.59)(19.41)$$

$$c = 0.03 \times 2(240.59)(19.41)$$

$$c = 289.20 \text{ lb sec}/\text{ft}$$

Put the value in Eq (A)

$$9062s + 289.20 + 240.59 = 0$$

Solution to the EOM for Damped free vibration is

$$u(t) = e^{-\delta\omega_n t} \left[ u(0) \cos(\omega_D t) + \frac{1}{\omega_D} \left[ \dot{u}(0) + \delta\omega_n u(0) \right] \sin(\omega_D t) \right]$$

$$\omega_D = 19.41 \text{ rad/sec}$$

$$u(t) = e^{-0.582t} \left[ 0.041 \times \cos(19.41t) + 0.00128 \times \sin(19.41t) \right]$$

$$F_s(t) = k \cdot u(t)$$

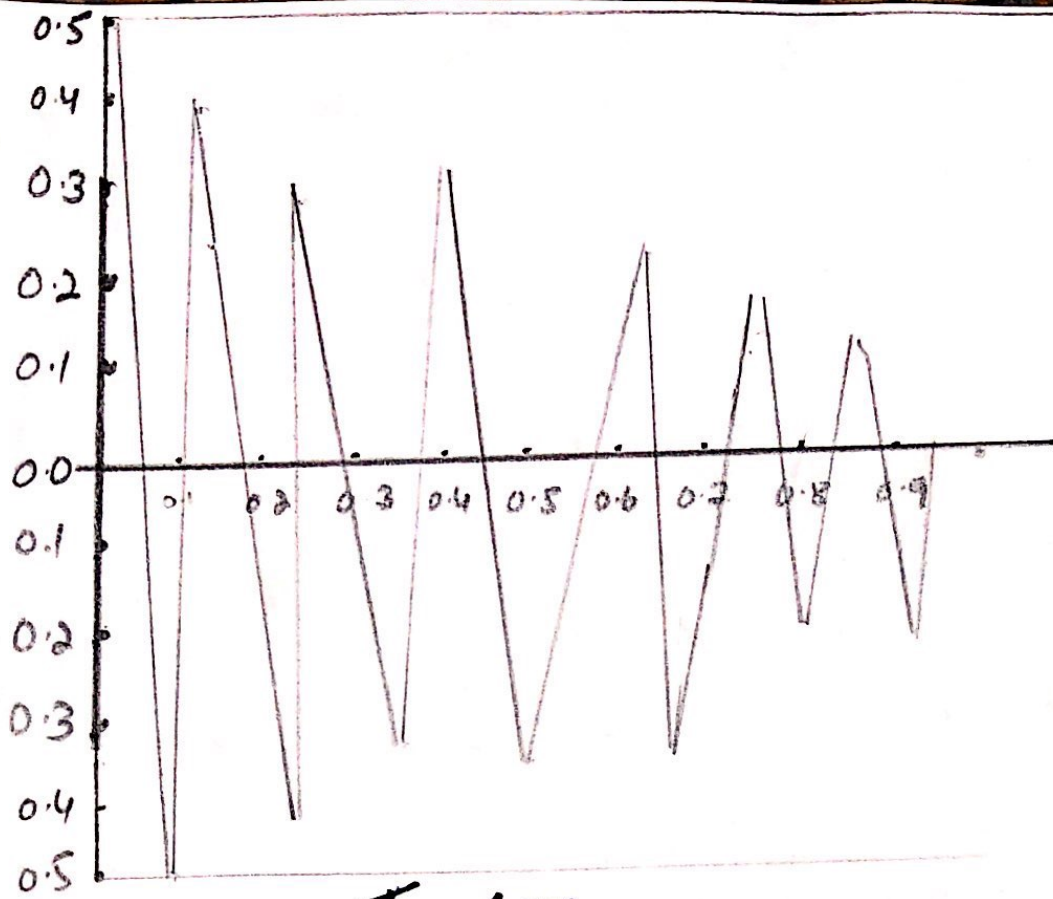
$$F_s(t) = 9062s \times u(t)$$

$$F_s(t) = e^{-0.582t} \left[ (9062s \times 0.041) \cos(19.41t) + 9062s \times 0.00128 \sin(19.41t) \right]$$

$$F_s(t) = e^{-0.582t} \left[ 3715.62 \cos(19.41t) + 113.28 \times \sin(19.41t) \right]$$

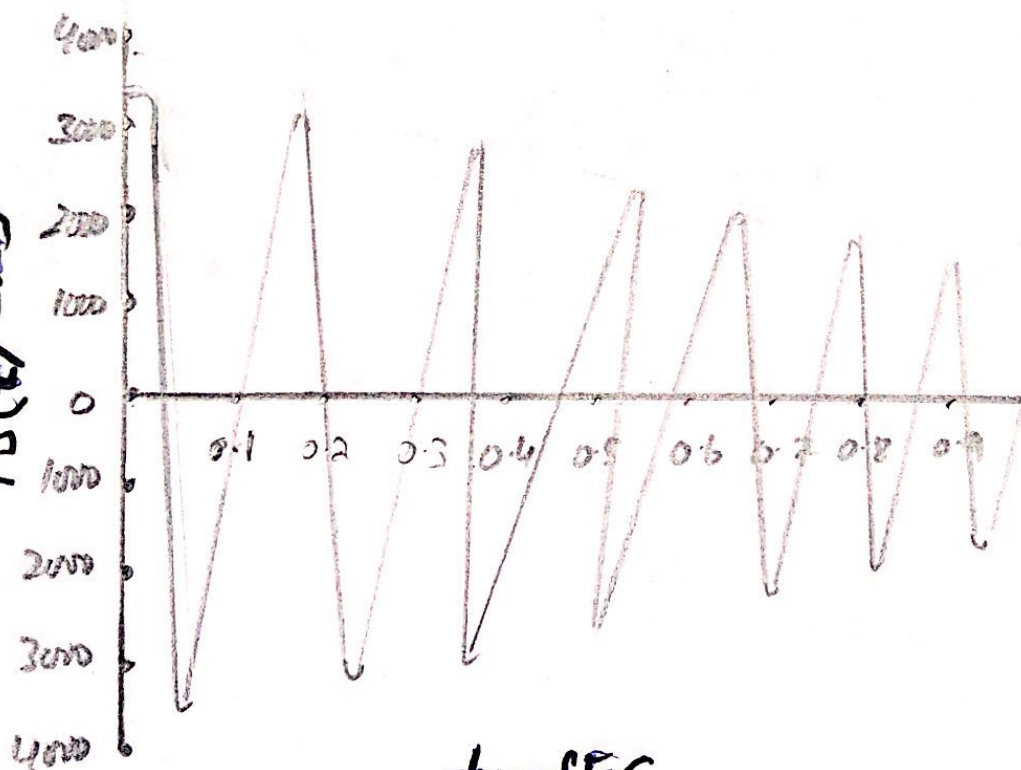


$U(t) - IN$



$t - SEC$

$F(t) - lb$



$t - SEC$

# ANSWER TO QUESTION # 3

## Given data :

Force = 60 kips

Displacement of tank =  $\frac{1D}{1000} = \frac{7747}{1000}$

$$= 7.747$$

Time taken to complete 7 cycles  
= 3.57 sec

Amplitude of displacement = 2.286 m  
= 0.9"

## Required :

- ① Damping Ratio
- ② Natural period of unimped vibration
- ③ Stiffness of structure
- ④ Weight of tank
- ⑤ Damping coefficient
- ⑥ Displacement amplitude to 0.5"  
Number of cycle of  
Reduce.

## SOLUTION:

Displacement of tank,  $u_1 = 7.747$

After, 7 cycles

$$J = 7, \quad 4J + 1 = u_8 = 0.9$$

### 1 → Damping Ratio:

$$J = \frac{1}{2\pi J} \ln \left( \frac{u_1}{u_{1+1}} \right)$$

$$7 = \frac{1}{2\pi 7} \ln \left( \frac{7.747}{0.9} \right)$$

$$J = 0.04889$$

$$J = 4.89\%$$

### 2 → Natural Period of Undamped Vibration = $T_n = ?$

As the 7 cycle of vibration are complete in 3.57 sec.

→ Time Req to complete one cycle,  $T_D = \frac{3.57}{7} = 0.51 \text{ sec}$

$$T_D = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{(1 - J^2)}$$



$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$T_D = \frac{T_n}{(1-\zeta^2)}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0489)^2}$$

$$\boxed{T_n = 0.51}$$

3 → STIFFNESS OF STRUCTURE,  $k = ?$

$$k = \frac{60 \times \cos 60^\circ}{7.747} \Rightarrow 3.872 \text{ k/m}$$

$$k = 3.9 \text{ k/m}$$

$$\boxed{k = 46920 \text{ lb/ft}}$$

4 → Weight of tank,  $w = ?$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{w/g}} \quad \because m = w/g$$

$$\omega_n = \sqrt{\frac{k \cdot g}{w}}$$

Taking Square b/s

$$\omega_n^2 = \frac{k \cdot g}{w}$$

$$\omega_n^2 = \frac{k \cdot g}{w}$$

$$W = \frac{k \times g}{\omega_n^2}$$

also;

$$\omega_n = \frac{2\pi}{T_n}$$

$$W = \frac{k \cdot g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = \frac{k \times g \times T_n^2}{4\pi^2}$$

$$= \left[ \frac{46920}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2} \right] \times \left[ \frac{(0.51 \text{ sec})^2}{4\pi} \right]$$

$$\Rightarrow 9953.93 \text{ lb}$$

$$\Rightarrow 9.95 \text{ k}$$

→ Damping Co-efficient, c = ?

$$\zeta = \frac{c}{2m\omega_n}$$

$$c = \zeta \times 2m\omega_n \Rightarrow \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = 0.0489 \times \frac{4 \times \pi}{2} \times \frac{9953.9}{32.2}$$

$$0.51$$

$$c = 371.71 \text{ lb} \cdot \text{sec} / \text{ft}$$

6 → NUMBER OF CYCLES TO REDUCE  
the Displacement AMPLITUDE  
TO 0.5", J=?

$$J = \frac{1}{2\pi \times 7} \ln \left( \frac{u_1}{u_2 + 1} \right)$$

$$J = \frac{1}{2\pi \times 0.00489} \ln \left( \frac{7.747}{0.5} \right)$$

$$J = 8.919$$

$$J = 9 \text{ cycles}$$

