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Module = 18.

ID = 6620

Subject = Signal & System.

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Date:

Question No 1A.

As we know that

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} \cdot dt.$$

Differentiating both sides with r.t ω

$$\frac{dX}{d\omega}(j\omega) = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t}\, dt$$

$$\frac{dX}{d\omega}(j\omega) = -jt \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$\frac{dX}{d\omega}(j\omega) = -jt\, \mathcal{F}\{x(t)\}$$

$$-jt\, x(t) \xleftrightarrow{\mathcal{F}} \frac{d}{dt} X(j\omega)$$

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Question 1b. $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

$X(z) = 2 - 4z^{-2} + 2z^{-3}$

$H(z) = 3 + 1z^{-1} + 2z^{-2}$.

Now

$Y_z = H(z) * X(z)$

$= (2 - 4z^{-2} + 2z^{-3})(3 + 1z^{-1} + 2z^{-2})$.

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-5}$

$\Rightarrow 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$

To find $y(x)$ use the delay property

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] + 2\delta[n-3]$

$- 6\delta[n-4] + 4\delta[n-5]$

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Question 2

$$f(x) \begin{cases} -\pi/2 & -\pi \geq x \leq 0 \\ \pi/2 & 0 \leq x \leq x. \end{cases}$$

Sol:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx.$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx\right]$$

$$= \frac{1}{2\pi}\int_{\pi}^{0} -\frac{\pi}{2}\, dx + \frac{\pi}{2}\int_{0}^{\pi} \frac{\pi}{2}\, dx$$

$$= \frac{1}{2\pi}\int -\frac{\pi}{2}\int_{-\pi}^{0} 1\cdot dx + \frac{\pi}{2}\int_{0}^{\pi} 1\, dx$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2} x \Big|_{-\pi}^{0} x \Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi}{2}(\pi) + \frac{\pi}{2}[\pi]\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi^2}{2} + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[\frac{0}{2}\right] \Rightarrow \; a \Rightarrow 0$$

now coefficient.

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f\cos nx\,dx.$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\cos nx\,dx.$$

$$= \frac{1}{\pi}\int \frac{\pi}{2}\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\frac{\sin nx}{n}\Big|_{0}^{\pi}$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\sin n(0) - \sin n(-\pi)\right].$$

$$+\frac{\pi}{2}\left[\sin n(\pi) - \sin n(0)\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$= \frac{1}{n\pi}(0).$$

$$\boxed{a_n = 0}$$

Now

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin x\,dx$$

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$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} \sin nx\,dx + \frac{\pi}{2}\int_{0}^{\pi} \sin nx\,dx.\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\,\frac{-\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\,\frac{-\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-1+\cos n.(\pi)\right] + \frac{\pi}{2}\left[-\cos n\pi + \cos n(0)\right]\right]$$

$$\frac{\pi}{2} = \frac{1}{n\pi}\left[-1\left[-1+\cos n(-\pi)\right] + 1\left[-\cos nx + 1\right]\right]$$

$$= \frac{1}{2n}\left[1-\cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2-2\cos n\pi\right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd} \end{cases}$$

$$\left\{ b_n = \frac{4}{2n} \right\}.$$

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \dots$$
$$+ b_1\sin x + b_2\sin 2x + b_3\sin 3x + \dots$$

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$f(x) = (0) + (0)\cos x + 0\cos(2x) + 0\cos 3x + \cdots$

$= \frac{4}{2}\sin x + (0)\sin^2 x + \frac{4}{3(2)}\sin 3x + \cdots\cdots$

$\{ \frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \cdots \}$.

Question No (3)

$X(z) = \frac{2z^2+2z}{(z^2+2z-3}$.

$x(z) = \frac{2z(z+1)}{z^2+2z-3}$.

$\frac{x(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$

$\frac{2(z+1)}{(z+3)(z-1)} = \frac{A}{(z+3)} + \frac{B}{(z-1)} \longrightarrow (i)$

$2(z+1) = A(z-1) + B(z+3) \longrightarrow (ii)$

put $z=1$ in (ii)

$2(1+1) = A(1-1) + B(1+4)$

$4 = 0 + 4B$

$B = 1$

Now put $z = -3$ in (ii)

$2(-3+1) = A(-3-1) + B(\cancel{-3}+\cancel{3})$

$2(-2) = A(-4) + 0$

$-4 = -4A$.

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$A = 1$ put in (1)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}.$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

Invers. z-transform

$$x[n] = u[3] + 1[-1]^{k}$$

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Question No 4.

$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$.

$G(s) = C(SI - A) + B + D.$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left[s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ 0-1 & s-0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$

$G(s) = \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{s^2+2s+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$

$= \frac{1}{s^2+2s+1} \begin{bmatrix} s & 2 \end{bmatrix}$

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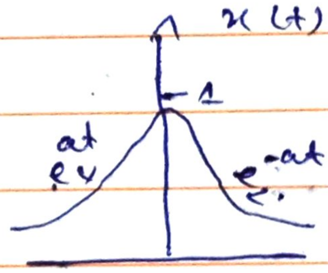
Question No (5) $u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$

Sol:-

The fourier transform of the given function $x(t)$ is given by

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt.$$



Note:

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t<0 \end{cases}$$

$\therefore$

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt.$$

$$X(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left.\frac{e^{(a-j\omega)t}}{a-j\omega}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{(a+j\omega)}\left[e^{-\infty} - e^{0}\right]$$

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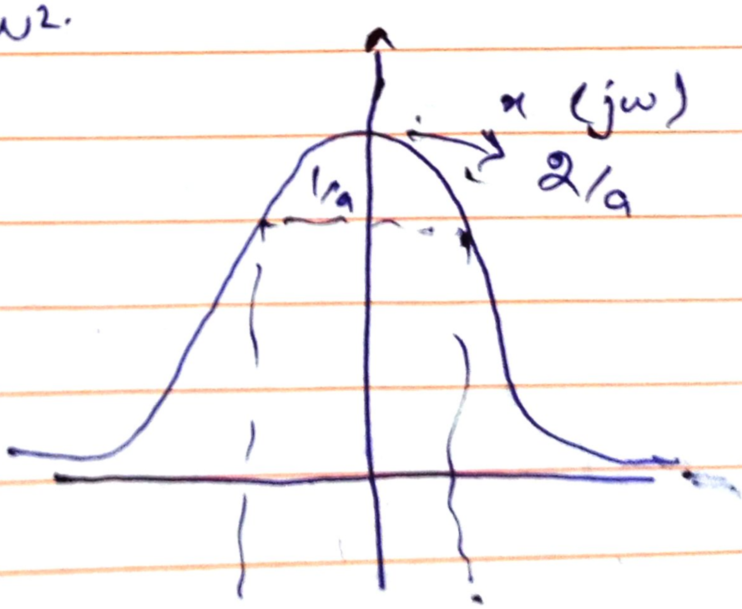
$$\frac{1}{(a-j\omega)}\left[1-0\right]-\frac{1}{(a+j\omega)}\left[0-1\right]$$

$$=\frac{1}{a-j\omega}+\frac{1}{a+j\omega}$$

$$=\frac{a+j\omega+a-j\omega}{a^2-(j\omega)^2}$$

$$(j\omega)=\frac{2a}{a^2+\omega^2}.$$



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