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(1)

QUESTION No # 1

The function $g(t)$ is defined by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

(a) State any point of discontinuity

(b) Find, if they exist

i) $\lim_{t \to 3} g$

Sol:- To check possibility of the discontinuity of the function is at $t = 0$ & 4

First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

$$= \lim_{h \to 0} 1 + h^2 + 2h$$

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(2) Apply Limits

$= 1 + 0^2 + 2(0)$

$= 1$

For L.H.L

$\lim_{h \to 0} g(1-h) = 2t + 3$

$= \lim_{h \to 0} 2(1-h) + 3$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L $= g(t) = 5$

→ Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L

(3) $\lim g(2+h) = \lim_{h \to 0} 2(2+h)+3$

$= \lim_{h \to 0} 2+2h+3$

Apply limits

$= 2+2(0)+3 \Rightarrow 5$

For L.H.L

$\lim_{h \to 0} g(2-h) = 12$

$g(4) = R.H.L \neq L.H.L$

Point of discontinuity is at $t=4$

(b) Find, if they exist

i) $\lim_{t \to 3} g$

For $g(t) = t^2$

R.H.L $\lim_{h \to 3} g(1+h) = \lim_{h \to 3} (1+h)^2$

$= \lim_{h \to 3} 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2(3) \Rightarrow 16$

(4)

L.H.L

$$\lim_{h \to 3} g(1-h) = \lim_{h \to 3} 2t+3$$

$$= \lim_{h \to 3} 2(1-h)+3$$

$$= \lim_{h \to 3} 2 - 2h + 3$$

Apply limit

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

R.H.L $\neq$ L.H.L (do not exist since L.H.L is -ve)

③ ⑤ QUESTION NO # 2

$Y(x) = x^2 + \sin x$

Since we know that the maclurian series is

$$Y(x) = Y(x_0) + Y'(x_0)(x - x_0) + \frac{Y''(x_0)(x-x_0)^2}{2!} + \cdots$$

Put $x_0 = 0$

$$Y(x) = Y(0) + (x-0)Y'(0) + \frac{(x-0)^2 Y''(0)}{2!} + \cdots$$

$$Y(x) = Y(0) + xY'(0) + \frac{x^2 Y''(0)}{2!} + \cdots \quad (1)$$

Now find

$Y(0) = ?$

$Y(x) = x^2 + \sin x$

$Y(0) = 0 + \sin 0$

$= 0 + 0$

$= 0$

$$\boxed{Y(0) = 0}$$

(b) $\cancel{(4)}$ $\cancel{(2)}$

$$y(x) = x^2 + \sin x$$

$$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$$

$$y'(x) = 2x + \cos x$$

$$y'(0) = 2(0) + (0)o$$

$$= 0 + 1$$

$$\boxed{y'(0) = 1}$$

Since $y'(x) = 2x + \cos x$

$$\frac{d}{dx} y'(x) = 2\frac{d*}{dx} x + \frac{d}{dx} \cos x$$

$$= 2 - \sin x$$

$$y''(x) = 2 - \sin x$$

$$y''(0) = 2 - \sin 0$$

$$= 2 - 0 = 2$$

$$\boxed{y''(0) = 2}$$

7 8 3

Now

$y''(x) = 2 - \sin x$

$$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$$

$$= 0 - \cos x$$

$$y'''(x) = 0 - \cos x$$

$$y'''(0) = -\cos 0$$

$$\boxed{y'''(0) = -1}$$

Put in eq 1

$$y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \dots$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \dots$$

$$= x + x^2 - \frac{x^3}{3!} + \dots$$

So

$$y(x) = x + x^2 - \frac{x^3}{3!} + \dots\dots$$

⑧ QUESTION NO # 3

Part → a) $1 + xy = x^2 + y^2$, find $y'' = ?$

Sol:- Given

$$1 + xy = x^2 + y^2$$

taking $\frac{d}{dx}$ on both sides

$$1 + \frac{d}{dx} \cdot x \frac{dy}{dx} = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$\Rightarrow 1 + (1)\left(\frac{dy}{dx}\right) = 2x + 2y\frac{dy}{dx}$$

$$1 + \frac{dy}{dx} = 2x + 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - 1$$

$$\frac{dy}{dx}(1-2y) = 2x-1$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x-1}{1-2y}$$

$$\Rightarrow \boxed{y' = \frac{2x-1}{1-2y}} \rightarrow ①$$

Diff again

(9)

$$\frac{d}{dx} y' = \frac{d}{dx}\left(\frac{2x-1}{1-2y}\right)$$

$$y'' = \frac{(1-2y)\frac{d}{dx}(2x-1) - (2x+1)\frac{d}{dx}(1-2y)}{(1-2y)^2}$$ use Quoitent rule

$$= \frac{(1-2y)(2) - (2x-1)(-2y')}{(1-2y)^2}$$

$$= \frac{(2-4y) - (2x-1)(-2y')}{(1-2y)^2}$$

From eq (1) $y' = \frac{2x-1}{1-2y}$

Put in above

$$y'' = \frac{(2-4y) - (2x-1)\left(-2\left(\frac{2x-1}{1-2y}\right)\right)}{(1-2y)^2}$$

$$= \frac{2\cancel{(1-2y)}}{(1-2y)^{\cancel{2}}} - \frac{(2x-1)(2x-1)(-2)}{(1-2y)^3}$$

$$\Rightarrow y'' \; \frac{2}{1-2y} - \frac{-2(2x-1)^2}{(1-2y)^3}$$ Ans.

Part → (b)

$$\ln(y) = \ln\left(x^3 (1+x)^9 e^{6x}\right)$$

$$= \ln\left(x^3 (1+x)^9\right) + \ln e^{(6x)}$$

$$= \ln x^3 + \ln(1+x)^9 + 6x$$

$$= 3\ln x + 9\ln(1+x) + 6x$$

Now

$$\frac{d \ln(y)}{dx} = \frac{d}{dx}\left(3\ln x + 9\ln(1+x) + 6x\right)$$

$$= 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}$$

$$= 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} + 6$$

$$\boxed{\frac{d \ln(y)}{dx} = \frac{3}{x} + \frac{9}{x+1} + 6}$$