Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Final – Term Examination summer2020
Date:26/9/2020

Course Code: MTH 203 **Course Title:** Differential equation
Prerequisite: CALCULUS **Instructor:** HIMAYATULLAH
Module: 3 **Program:** BEE **Total Marks:** 80 **Time Allowed:** _____

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.		. Estimate general solution of $4y'' - 20y' + 25y = 0$.	Marks 15 PLO1 C2
Q2		Estimate the general solution of $y' = (x+2)y^2$	Marks 15
Q3		Calculat the initial value problem $y'' + 2y' + y = 0$ $y(0) = 4.\ \ y'(0) = -6.$	Marks 10 PLO2 C4
Q4		Analyze the general solution of $x^2y'' + 3xy' + y = 0$.	Marks 10 PLO2 C4
Q5		**Examine** the method of undetermined coefficient method for $y'' + y' - 6y = 6x^3 - 3x^2 + 12x$.	Marks 10 PLO2 C4
Q6		**Examine** the method of variation of parameters for $y'' - 4y' + 4y = x^2e^{2x}$	Marks 10 PLO2 C4
Q7		Identify an ODE $y'' + ay' + by = 0$ for the basis 1, e^{-3x}..	Marks 10 C4 PLO2

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Course Differential Equation.

Q1 Estimate general solution of
$4y'' - 20y' + 25y = 0$.

Sol:-

$$4y'' - 20y' + 25y = 0$$

$$y'' + ay' + by = 0$$

$$4y'' - 20y' + 25y = 0$$

÷ing by 4.

$$y'' - 5y' + 25/4\, y = 0$$

$$a = -5, \quad b = 25/4$$

Auxilary equation

$$\lambda^2 + a\lambda + b = 0$$

$$\lambda^2 - 5\lambda + 25/4 = 0$$

$$(\lambda - 5/2)^2 = 0$$

$$(\lambda + 5/2)(\lambda - 5/2) = 0$$

$$(\lambda + 5/2), (\lambda - 5/2)$$

$$\lambda = 5/2, \quad \lambda = -5/2$$

Root are Real & distinct

$$y = C_1 e^{-1x} + C_2 e^{1x}$$

$$y = C_1 e^{-5/2x} + C_2 e^{5/2x}$$ Ans

Q3. Calculate the initial value problem

$y' + 2y + 6 = 0$

$y(0) = 4, \quad y'(0) = -6$

Sol:- $y' + 2y + 6 = 0$

$y(0) = 4, \quad y'(0) = -6$

Homogenous equation

$y'' + 2y' + y = 0$

Auxilary equation

$\lambda^2 + 2\lambda + 1 = 0$

$\lambda^2 + \lambda + \lambda + 1 = 0$

$\lambda(\lambda+1) + 1(\lambda+1)$

Root are real & equal

$y = c_1 e^{-x} + c_2 x e^{-x}$

$y' = c_1 e^{-x} + c_2 e^{-x} - x e^{-x}$

When $y = 4, \quad x = 0$

$y = c_1 e^{-x} + c_2 x e^{-x}$

$4 = c_1 e^{0} + c_2 x e^{0}$

$\boxed{4 = c_1}$ — (1)

When $x = 0, \quad y = -6$

$-6 = c_1 e^{0} + c_2 e^{0} - 0 e^{0}$

$\boxed{-6 = c_1 + c_2}$ — (2)

Adding ① & ②

$$4 = c_1$$
$$-6 = c_1 + c_2$$
$$-10 = c_2$$

$$\boxed{c_2 = -10}$$

~~$y = (c_1 - 10x)$~~

— x — x — x — x — x — x —

Q4. Analyze the general solution
$x^2y'' + 3xy' + y = 0$

Sol:- $x^2y'' + 3xy' + y = 0$

$a = 3, \quad b = 1$

$m^2 + (a-1)m + b = 0$

$m^2 + (3-1)m + 1 = 0$

$m^2 + (2)m + 1 = 0$
$m^2 + 2m + 1 = 0$
$m^2 + m \cdot m + 1 = 0$
$m(m+1) + 1(m+1) = 0$

~~$m(m+1)$~~
$(m+1)(m+1) = 0$

$m = -1, \quad m = -1$
Root are Real & equal

$y = (c_1 + c_2 \ln x) x^m$

$y = (c_1 + c_2 \ln x) x^{-1}$

— x — x — x — x — x — x — x — x —

Q5 Examine the method of undetermined coefficient method for $y'' + y - 6y = 6x^3 - 3x^2 + 12x$

Sol: $y'' + y - 6y = 6x^3 - 3x^2 + 12x$

for Home equation

$y'' + y' - 6 = 0$

Auxilary equation

$\lambda^2 + a\lambda + by = 0$

$\lambda^2 + \lambda - 6 = 0$

$\lambda^2 + 3\lambda - 2\lambda - 6 = 0$

$\lambda(\lambda+3) - 2(\lambda+3) = 0$

$\lambda + 3 = 0, \quad \lambda - 2 = 0$

$\lambda = -3, \quad \lambda = 2$

Root are Real & distan

$y = c_1 e^{-3x} + c_2 e^{2x}$

$y_p = k_3 x^3 + k_2 x^2 + k_1 x + k_0$

$y'_p = 3k_3 x^3 + 2k x^2 + k_1$

$y'_p = 6k_3 x - 2k_2$ put in 1

$6k_3x - 2k_2 + 3k_3x^2 + 2k_2x + k_1 - 6k_3x^3 - 6k_2x^2 - 6k_1x - 6k_0$

$= 6x^3 - 3x^2 + 12x$

Comparing:

$-6k_3 = 6$

$k_3 = -1$

$-6k_2 + 3k = -3$

$-6k_2 + 3(-1) = -3$

$6k_2 - 3 = -3$

$-6k_2 = -3 + 3$

$-6k_2 = 0$

$k_2 = 0$

$6k_3 + 2k_2 + k = 12$

$6(-1) + 2(0) + k_1 = 12$

$-6 + k_1 = 12$

$k_1 = -2$

$-2k_2 + k_1 + k_0 = 0$

$-2(0) - 2 + k_0 = 0$

$k_0 = 2$

Hence

$y = y_n + y_e$

$y = c_1e^{-3x} + c_2e^{2x} - x^3 + 0x^2 - 2x + 2$

$y = c_1e^{-3x} + c_2e^{2x} - x^3 - 2x + 2$

Q6:- Examine the method of variation of parameter's for

$$y'' - 4y' + 4y = x^2 e^{2x}.$$

Sol:- $y'' - 4y + 4y' = x^2 e^{2x}.$

for equation.

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$$

$$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda = 2, \quad \lambda = 2.$$

Root are Real & equal

$$y = (c_1 + c_2 x) e^{2x}$$

$$y_1 = c_1 e^{2x} + c_2 x e^{2x}$$

$$y_1 = e^{2x}, \quad y_2 = x e^{2x}$$

$$y_1' = 2e^{2x}, \quad y_2' = e^{2x} + 2ex^{2x}.$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2ex^2 x \end{vmatrix}$$

$$W = e^{4x} + 2xe^{4x} - 2xe^{4x}$$

$$W = e^{4x}.$$

$$y_p = -y_1 \int \frac{y_2 r(x)}{W} + y_2 \int \frac{y_1 r(x)}{W}$$

$$y_p = -e^{-2x} \int \frac{xe^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx + xe^{2x} \int \frac{e^{2x} \cdot x^2 e^{2x}}{e^{4x}} dx$$

$$y_p = -e^{2x} \int \frac{x^3 e^{4x}}{e^{4x}} + xe^{2x} \int \frac{x^2 e^{4x}}{e^{4x}} dx$$

$$y_p = -e^{2x} \int x^2 dx + xe^{2x} \int x^2 dx$$

$$y_p = -e^{2x} \cdot \frac{x^4}{4} + xe^{2x} \cdot \frac{x^3}{3}$$

So,

$$y = y_x + y_p$$

$$\boxed{y = c_1 e^{2x} + c_2 xe^{2x} - e^{2x} \frac{x^4}{4} + xe^{2x} \frac{x^3}{3}}$$

Q 7: Identify an ode $y'' + ay + by = 0$ for the basis $1, e^{-3x}$

Sol:-

$y'' + ay + by = 0$,

basis = $1, e^{-3x}$

$1, e^{-3x}$

$y = c_1 e^{x} + c_2 e^{-3x}$

$\lambda = 0, \quad \lambda = -3$

$\lambda - 0 = 0 \quad \lambda + 3 = 0$

$\lambda(\lambda + 3) = 0$

$\lambda^2 + 3\lambda = 0$

$a = 3, \quad b = 0$

$y'' - 3y' + 0y = 0$

$y'' - 3y' = 0$

— x — x — x — x — x — x — x —

Q2. Estimate the general solution of $y' = (x+2)y^2$

Sol:- $y' = (x+2)y^2$.

As we know that

$$y' = \frac{dy}{dx}$$

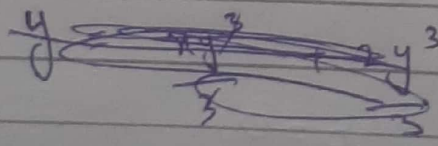
So,

$$dy/dx = (x+2)y^2$$

integrating both sides w.r.t. y.

$$\int dy/dx = \int (x+2)y^2 \, dy$$

$$\int d/dx\, y = \int xy^2 + y + \int 2y^2 \, dy$$



$$y = \frac{xy^3}{3} + \frac{2y^3}{3} + C$$

Ans