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Subject # Multivariate Calculus

Final- term

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1) Evaluate

$$\int_0^5 \int_0^x x(x+3x)\,dy\,dx$$

Solution:-

$$\int_0^5 \int_0^x x(x+3x)\,dy\,dx$$

$$= \int_0^5 \int_0^x (x^2+3x^2)\,dy\,dx$$

$$= \int_0^5 \left[\int_0^x (x^2 + 3x^2) dy \right] dx$$

$$= \int_0^5 \left[\int_0^x (4x^2 dy \right] dx$$

$$= \int_0^5 4x^2 (y)_0^x \, dx$$

$$= \int_0^5 4x^2 (x - 0) dx$$

$$= \int_0^5 4x^3 dx \qquad \because \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$= 4 \left[\frac{x^4}{4} \right]_0^5 = \frac{4}{4} \left[(5)^4 - 0 \right]$$

$$= (5)^4 = 625$$

$$= \int_0^5 \int_0^x x(x + 3x) dy \, dx = 625$$

2) Evaluate

$$\int_1^4 \int_0^3 xy + x^3y^3) \, dy \, dx$$

Solution:-

First, we integrate with respect to y.

$$= \int_1^4 \int_0^3 (xy + x^3y^3) \, dy \, dx$$

$$= \int_1^4 \left[\frac{xy^2}{2} + \frac{x^3y^4}{4} \right]_0^3 dx$$

$$= \int_1^4 \left(\left[\frac{x(3)^2}{2} + \frac{x^3(3)^4}{4} \right] - \left[\frac{x(0)^2}{2} + \frac{x^3(0)^4}{4} \right] \right) dx$$

$$= \int_1^4 \left(\frac{9x}{2} + \frac{81x^3}{4} \right) dx$$

then we integrate with respect to x

$$= \int_1^4 \left(\frac{9x}{2} + \frac{81x^3}{4} \right) dx$$

$$= \left[\frac{9x^2}{4} + \frac{81x^4}{16} \right]_1^4$$

$$= \left[\frac{9(4)^2}{4} + \frac{81(4)^4}{16} \right] - \left[\frac{9(1)^2}{4} + \frac{81(1)^4}{16} \right]$$

$$= \frac{21195}{16}$$

$$= 1324.6875$$

3) Find partial derivatives w.r.t r and s

$$f(r, s) = r \cdot \ln(r^3 + s^2)$$

Solution:-

We first find the partial derivative with respect to r. We will use Product rule and chain rule.

$$= \frac{\partial f}{\partial r} = \frac{\partial}{\partial r}\left(r \cdot \ln(r^3+s^2)\right)$$

$$= \frac{\partial}{\partial r}(r)\ln(r^3+s^2) + r\frac{\partial}{\partial r}\ln(r^3+s^2)$$

$$= (1)\ln(r^3+s^2) + r \cdot \frac{1}{(r^3+s^2)}\frac{\partial}{\partial r}(r^3+s^2)$$

$$= \ln(r^3+s^2) + r \cdot \frac{1}{(r^3+s^2)}(3r^2)$$

$$= \ln(r^3+s^2) + \frac{3r^3}{(r^3+s^2)}$$

$$= \frac{\partial r}{\partial r} = \ln(r^3+s^2) + \frac{3r^3}{(r^3+s^2)}$$

Then we find the partial derivative with respect to s.

$$\frac{\partial f}{\partial s} = \frac{\partial}{\partial s}\left(r \cdot \ln(r^3+s^2)\right)$$

$$= r\frac{\partial}{\partial s}\ln(r^3+s^2)$$

$$= r \cdot \frac{1}{(r^3+s^2)}\frac{\partial}{\partial s}(r^3+s^2)$$

$$= r \cdot \frac{1}{(r^3+s^2)} (2s)$$

$$= \frac{2rs}{(r^3+s^2)}$$

$$= \frac{\partial f}{\partial s} = \frac{2rs}{(r^3+s^2)}$$

4) Finding Partial derivatives w.r.t "X" $F(x, y, z) = xy^2z^4 + 3yz^2$

Solution:-

Solve for x, $F = xy^2z^4 + 3yz^2$

: $x = \frac{F - 3yz^2}{y^2z^4}$; $y \neq 0, z \neq 0$

$= F = xy^2z^4 + 3yz^2$

$= xy^2z^4 + 3yz^2 = F$

subtract $3yz^2$ from b/s

$= xy^2z^4 + 3yz^2 - 3yz^2 = F - 3yz^2$

$= xy^2z^4 = F - 3yz^2$

Divide b/s by y^2z^4 ; $y \neq 0$; $z \neq 0$

$= \frac{xy^2z^4}{y^2z^4} = \frac{F}{y^2z^4} - \frac{3yz^2}{y^2z^4}$; $y \neq 0$, $z \neq 0$

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$$x = \frac{F - 3yz^2}{y^2 z^4} ; \; y \neq 0, z \neq 0$$

5) Find the value of x and y.

$8x - y = -1, \; 7x - y = -2$

Solution:-

Consider the given set of equation is,

$8x - y = -1$

$7x - y = -2$

Consider first expression is written as;

$8x + 1 = y$

Subtitute above the second equation as,

$7x - (8x + 1) = -2$

$= 7x - 8x - 1 = -2$

$= -x = -2 + 1$

$= -x = -1$

$= x = 1$

Substitute $x = 1$ in the first equation as,

$\cancel{\Rightarrow}$ $8(1) - y = -1$

$8 - y = -1$

$-y = -1 - 8$

$y = 9$

Thus, the value of $x = 1$ and $y = 9$