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SUBJECT: PROBABILITY AND STATISTICS

SEMESTER: 3RD

PROGRAMME: BS (SOFTWARE ENGINEERING)

Date:

Q1) A man throws -----?

- 1) The sum is even
- 2) The sum is greater than 8
- 3) The two dice had the same outcome

Sol:-

$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

Let

$A = \{ \text{the sum is 10} \}$
 $B = \{ \text{The sum is even} \}$
 $C = \{ \text{The sum is greater than 8} \}$
 $D = \{ \text{The dice had same outcome} \}$

$A = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$
 $B = \{ (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5) \}$
 $C = \{ (1,6), (2,5), (2,6), (3,4), (3,5), (3,6), (4,3), (4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6) \}$
 $D = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \}$

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$$A \cap B = \{(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)\}$$

$$A \cap C = \{(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)\}$$

$$A \cap D = \emptyset$$

$$P(A) = \frac{6}{36}, P(B) = \frac{18}{36}, P(C) = \frac{21}{36}, P(D) = \frac{6}{36}$$

$$P(A \cap B) = \frac{6}{36}, P(A \cap C) = \frac{6}{36} \text{ and } P(A \cap D) = 0$$

Hence

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{6}{36} \times \frac{36}{21} = \frac{2}{7}$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0 \times 36}{6} = 0$$

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Q2 Show that in a single throw of two dice -----?

Ans) Sum of 2 has 1 way 1, 1
Sum of 3 has 2 ways 1, 2 & 2, 1
Sum of 4 has 3 ways 1, 3; 2, 3, 1
5 has 4 ways
6 has 5 ways

8 has 5 ways (Symmetry)
9 has 4 ways

10 has 3 ways
11 has 2 ways
12 has 1 way

They are 15/36 for each side with a sum of 30/36 which leaves a $\frac{6}{36} = \frac{1}{6}$ probability for a sum of 7

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Q3) A and B play a game --- ?

Sol:-

Given that $P = \frac{2}{3}$ $n = 8$

$$q = 1 - P \\ = 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let x denotes the number of games won by A.

$$\begin{aligned} \text{i) } P(x=4) &= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 \\ &= \frac{1120}{6561} \\ &= 0.1707 \end{aligned}$$

$$\begin{aligned} \text{ii) } P(x \geq 4) &= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x} \end{aligned}$$

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$$= 1 - \left[\binom{8}{3} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right]$$

$$= 1 - \frac{1}{6561} [1 + 16 + 112 + 448]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

iii) $P(3 \leq x \leq 6)$

$$\sum_{n=3}^6 \binom{8}{n} \left(\frac{2}{3}\right)^n \left(\frac{1}{3}\right)^{8-n}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5$$

$$+ \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

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$$= \frac{8}{(3)^4} [56 + 140 + 224 + 224]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = 0.7852$$

Ans

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Q4) Let C_1, C_2, \dots, C_n ?

Sol:-

C_i 's form a partition of sample space,

Applying law of total probability for $A \cap B$:

$$\begin{aligned} P(A \cap B) &= \sum_{i=1}^n P(A \cap B | C_i) P(C_i) \\ &= \sum_{i=1}^n P(A | C_i) P(B | C_i) P(C_i) \quad (A, B \text{ conditionally independent}) \end{aligned}$$

$$= \sum_{i=1}^n P(A | C_i) P(B) P(C_i) \quad (B \text{ is independent of all } C_i \text{'s})$$

$$\begin{aligned} &= P(B) \sum_{i=1}^n P(A | C_i) P(C_i) \\ &= P(B) P(A) \quad (\text{Law of Total Probability}) \end{aligned}$$

Ans.

Date:

Q5 Derive Binominal Distribution -----?

Binominal Probability Distribution:

A binominal experiment is a statistical experiment that has the following properties.

- The experiment consists of repeated trials
- The trials are independent that is the outcome one trial does not affect the outcome of other trials.
- The probability of success, denoted by P is the same on every trial.

Mean and Variance:

$$N_n = n(P) \rightarrow \text{Mean (expected value)}$$

$$\sigma^2 = n(P)(1-P) \rightarrow \text{variance.}$$

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Q6 Differentiate between Binominal --?

Binominal Frequency Distribution:-

If the binominal probability distribution is multiplied by N , the number of experiments or sets, the resulting distribution is known as the binominal frequency distribution.

Formulae:-

$$N \binom{n}{r} p^r q^{n-r}$$

Binominal Distribution Formulae:-

$$P(X=r) f(r) = {}^n C_r p^r q^{n-r} \quad n=0,1,2,\dots$$

where

n = The number of sample size

r = The number of success

p = The probability of success

q = The probability of failure

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Q7) Below you will find ---- ?

Sol:-

Measure	Data set A	B	C	D
Coefficient of variation	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	$CV = 6.7$	$CV = 18.3$	$CV = 10$	$CV = 60$