(1)

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In : 7718

Section : B

Subject : Earthquake engineering

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Q1 (2)

Sol. ID = 7718

Given Data

$E = 29000$ ksi

$I = 150\ in^4$

$S_{st} = 7718$ lb

The general EOM for SDOF system

$$Ku + Cu + Mu = P(t)$$

-> In our case system is undamped $(c=0)$ undergoing free vibration $P(t) = 0$

-> Hence general EOM become $Ku + Mu = 0$ -> (1)

Now $K = \dfrac{3EI}{L^3} = \dfrac{3 \times 29000 \frac{K}{in^2} \times 150\ in^4}{(10 \times 12)^3}$

$K = 7.55\ k/in$

$$\boxed{K = 90625\ lb/ft}$$

$$m = \frac{7718\ lb\ sec^2}{32.2\ ft}$$

$$\boxed{\Rightarrow m = 239.689\ slug}$$

$$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90625}{239.68}}$$

~~$W_n = 20.01$ rad/sec~~ | $W_n = 19.445$ rad/sec

$$T_n = \frac{2\pi}{W_n} = \frac{2\pi}{20.01} = 0.314$$

$T_n = 0.314$ sec

→ Substituting the corresponding value in eq (1)

$$K\ddot{u} + M\ddot{u} = 0$$

$$90625u + 239.689\ddot{u} = 0$$

where 'k' is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free ~~va~~ vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$\because$ $u(0) = \frac{1}{2} = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$ ft

and $\dot{u}(0) = 0$

$$u(t) = \left(\frac{1}{24}\right)\left(\cos(~~20.01~~\ 19.44)\right) + 0$$

$$= \left(\frac{1}{24}\right)\left(\cos(~~20.01~~\ 19.44)\right)$$

Equivalent static force at any time 't' is

$$f_s(t) = k\,u(t) = \frac{90625 \times \cos(~~20.01~~\ 19.44)}{24}$$

$$f_{(s)}(t) = 3776.04 \cos(~~20.01~~\ 19.44)$$

$$= 3776.04\,(0.9429)$$

$$= ~~3547.967~~$$

$$= 3560.76$$

Amplitude of dynamic displacement

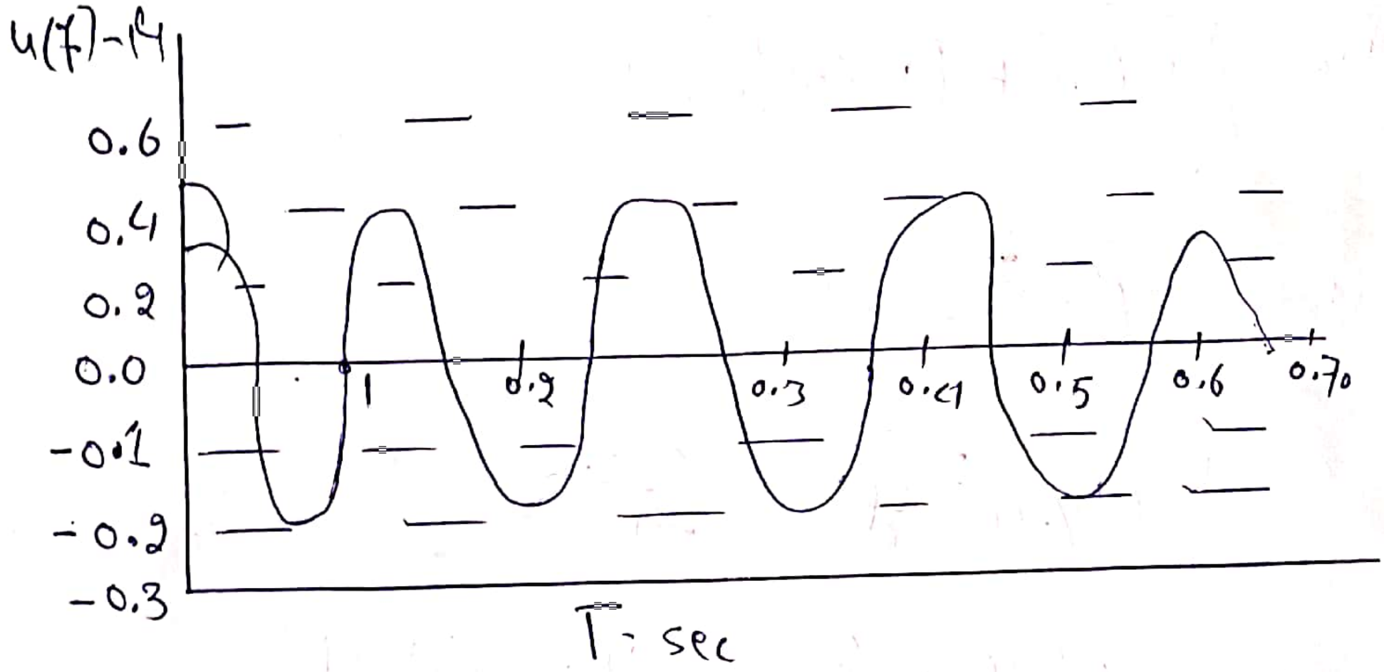
$$u_0 = \sqrt{u(0)^2 + \left(\dot{u}\cos/\omega_n\right)}$$

$$u_0 = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

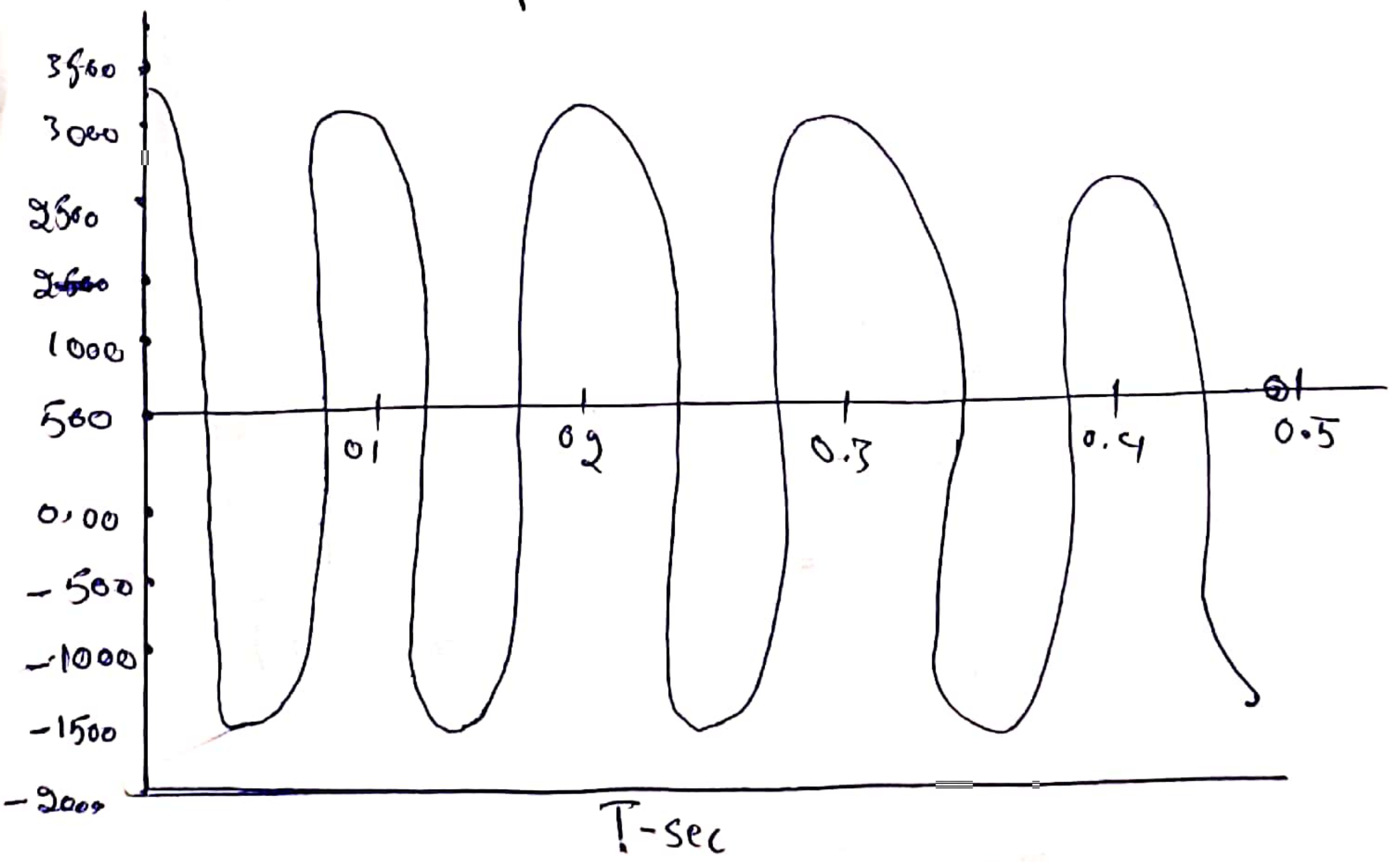
$$= \sqrt{(1/24)^2 \, ft} = \frac{1}{24} ft$$

$$Ku_0 = 90625 * \frac{1}{24} = 3776.042 \, lb$$

Undamped Free Vibration



Undamped Free Vibration



Q2

Soln

E = 29000 ksi

I = 150 in^4

δ_{st} = 7778 lb

Take ζ = 5%

EOM for damped free vibration

$Ku + C\dot{u} + m\ddot{u} = 0 \rightarrow (1)$

It is known from equation 1

$K = 90625$ lb/ft

$m = 239.689$ lb sec^2/ft

$C = \zeta \times 2m\omega_n$

$C = 0.05 \times 2(239.689)(19.445)$

$C = 0.05 \times 9321.505$

$C = 466.07$ lb sec/ft

By substituting values of k, c and m in eq (1)

$$90625u + 466.07u + 239.689u = 0$$

Solution to EOM for damped free vibration is:

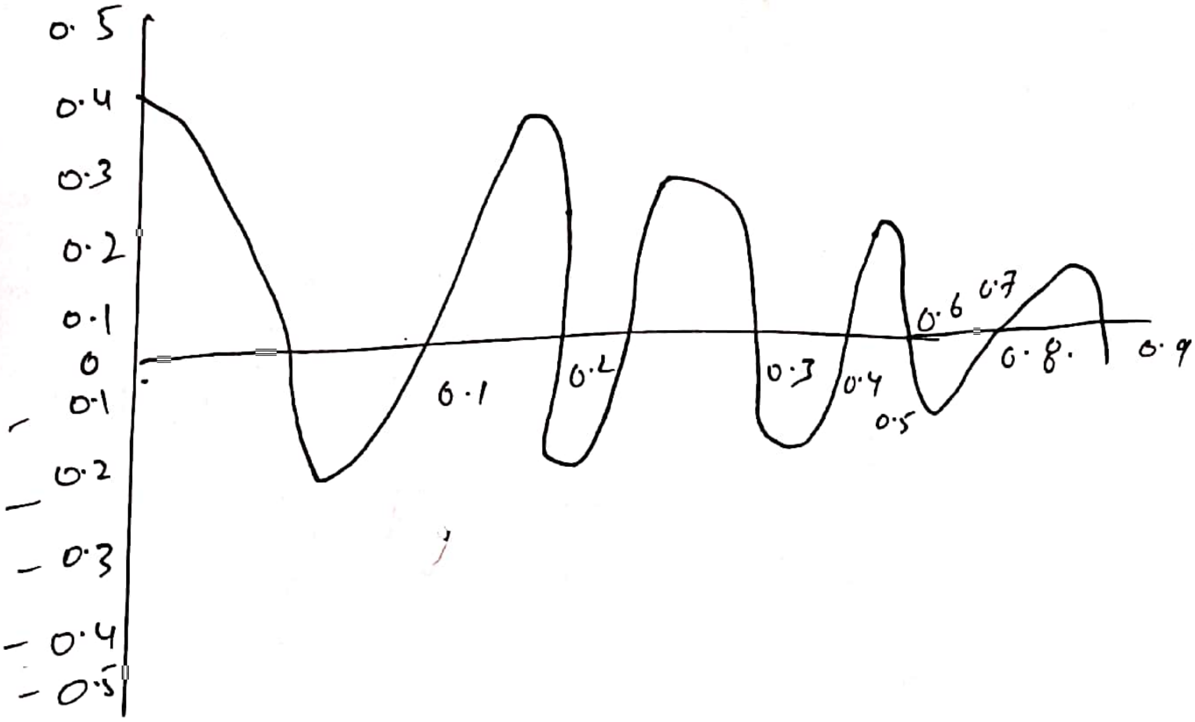
$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[u(0)\zeta\omega_n\right]\sin(\omega_D t)\right]$$

$$\Rightarrow \omega_D = \sqrt{\frac{k}{m}} = \sqrt{\frac{~~239.68~~}{}} \quad \sqrt{\frac{90625}{239.689}}$$

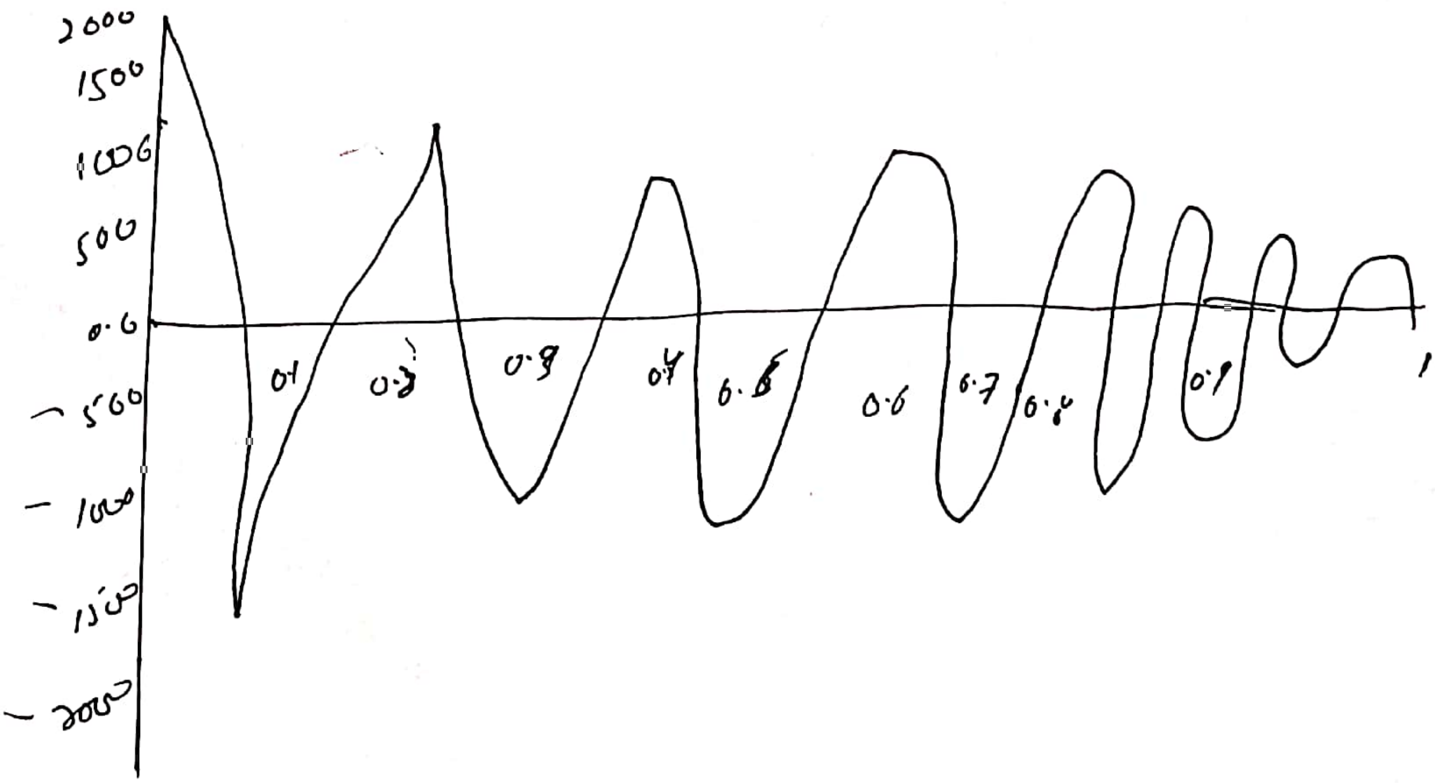
$$\Rightarrow 19.44 \text{ rad/sec}$$

$$\Rightarrow u(t) = e^{-0.05\times19.44t}\left[\frac{1}{24}\times\cos(19.44t) + \frac{1}{19.44}\times\left[0 + \frac{1}{24}\times0.05\times19.44 \times\sin(19.44\right]\right]$$

Damped Free vibration



Damped Free vibration



Q3

Sol. Given Data

Force = 60 kips

$u_1 = \frac{7718}{1000} = 7.718$ in

After, j = 7 (cycles)

Completed = 3.57 sec

$u_{j+1} = 2.286$ cm $= 0.9$ in

Ignore the verticle vibration

Required

(a) Damping ratios

(b) Natural period of undamped vibration

(c) Stifness of structures

(d) Weight of tank

(e) Damping co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5"

Sol

(a) ξ = Damping ratio = ?

As $j = \frac{1}{2\pi\xi} \ln\left[\frac{u_r}{u_{j+1}}\right]$

By putting values

$7 = \frac{1}{2(3.14)\xi} \ln\left[\frac{7.718}{0.9}\right]$

$\xi (7 \times 2 \times 3.14) = 2.148$

$\xi (43.96) = 2.148$

$\xi = \frac{2.148}{43.96}$

$= 0.0488$

$\boxed{\xi = 4.88\%}$

(c) (B)

$T_n = ?$

As "seven" cycles are completed in "3.57" sec

Thus time required to complete one

cycle $= 7/3.57 = 1.96$ sec

$T_D = 1.96$ sec

Now $\omega_D = \omega_n \sqrt{(1 - \zeta^2)}$

$\Rightarrow 2\pi/\omega_D = \frac{2\pi}{\omega_n} (\sqrt{1 - \zeta^2})$

As, $T_D = T_n / \sqrt{1 - \zeta^2}$

$\Rightarrow T_n = T_D (\sqrt{1 - \zeta^2})$

$= 1.96 (\sqrt{1 - (0.0462)^2})$

$T_n = 1.957$ "Natural period of undamped vibration"

(c) Stiffness of structure, "K = ?"

As $K = \frac{F \cdot \cos\theta}{2}$

$K = \frac{60 \cdot \cos(60)}{2}$ $\quad F = 60$ kips, $\theta = 60°$

$K = 15$ k/in

$K = 18000$ lb/ft

(d) Weight of Tank W = ?

As $\omega_n = \sqrt{K/m} = \sqrt{K/w/g}$

$= \sqrt{K \cdot g / W}$

$\Rightarrow \omega_n^2 = K \cdot g / W \Rightarrow (W = K \cdot g / \omega_n^2)$

By putting values of $\omega_n = 2\pi / T_n$

$W = K \cdot g / (4\pi^2 / T_n^2) = K \cdot g \left(\frac{T_n^2}{4\pi^2} \right)$

$W = \frac{18000 \text{ lb}}{\text{ft}} \cdot \frac{32.2 \text{ ft}}{\text{sec}^2} \left(\frac{(1.957)^2}{4(3.14)^2} \right)$

(15)

$W = 56284.75\ lb = 56.284\ klb$

'e' Damping Co-efficient "c = ?"

It is know that $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow c = \zeta(2m\omega_n) = \zeta(2m(2\pi/T_n))$

By putting value

$c = 0.0462\left(2\left(\frac{56284}{32.2}\right)\right)\left(2(3.14)\right)$

$c = 518.286\ lb\ sec/ft$

'f' No of cycles to reduce displacement altitude from "6.872 in to 0.5 in"

$j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0462)} \ln\left[\frac{7.718}{0.9}\right]$$

$j = 7.40$ cycles