

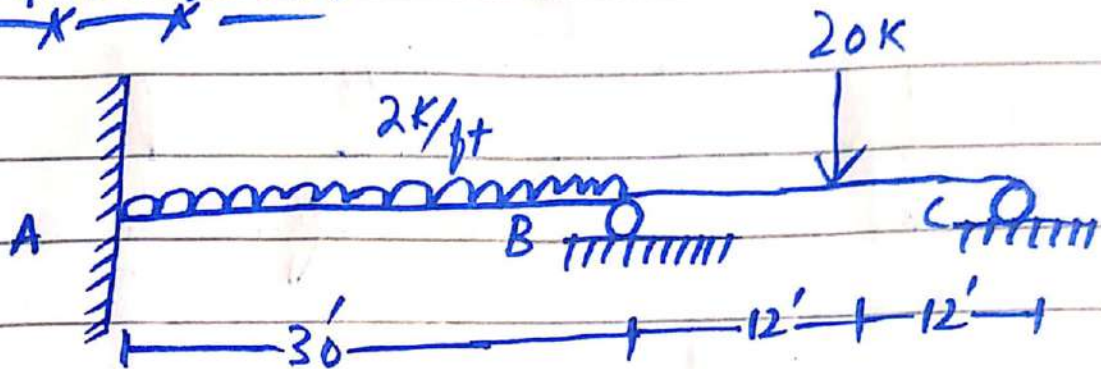
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ID=7693

PAPER MID SUMMER= STRUCTURE
ANALYSIS 2

Prob# 01

Given data:-

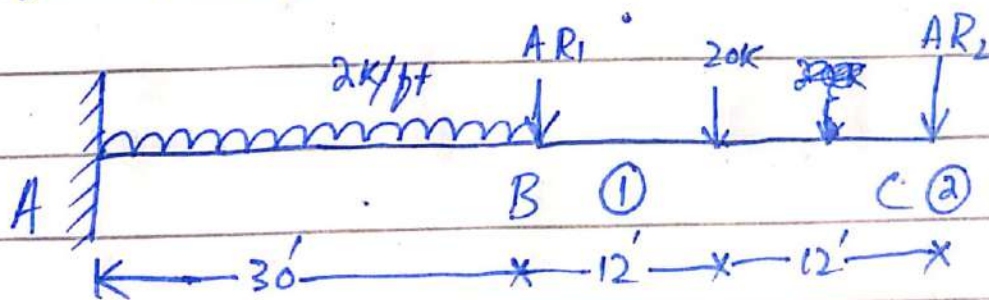


$EI = \text{Constant}$

Required Data: S.F.D = ?
B.M.D = ?

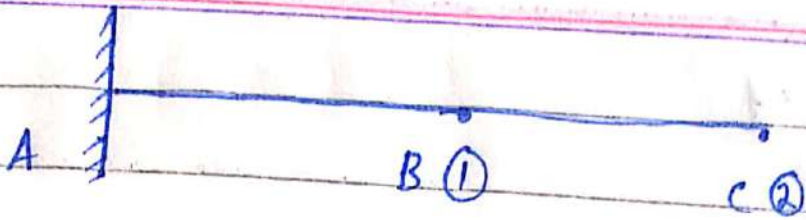
Solution:

$$S.I = 8 - 3m = 5 - 3(1) = 2^\circ$$

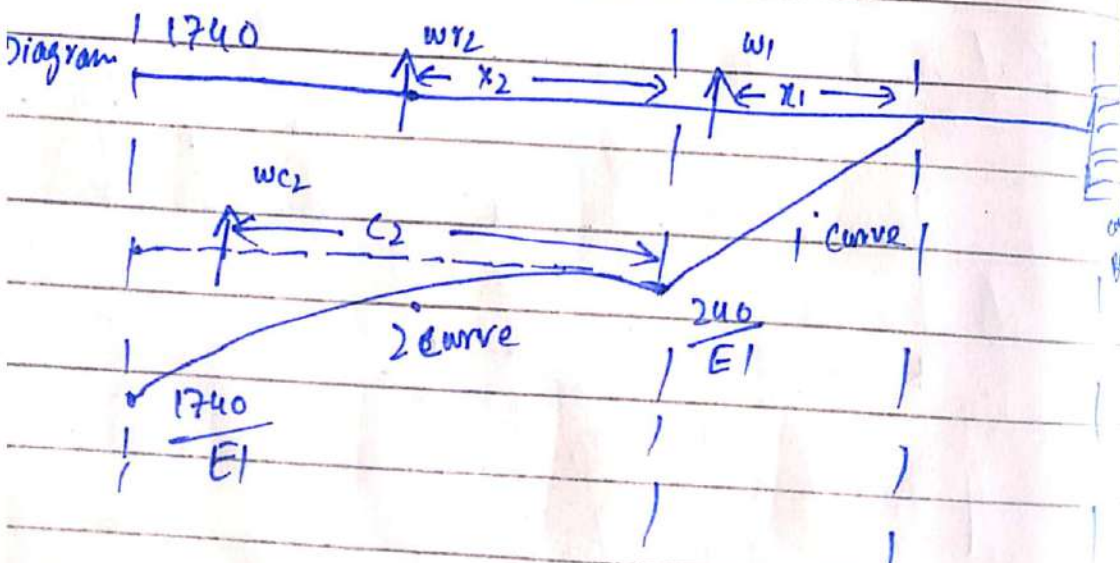
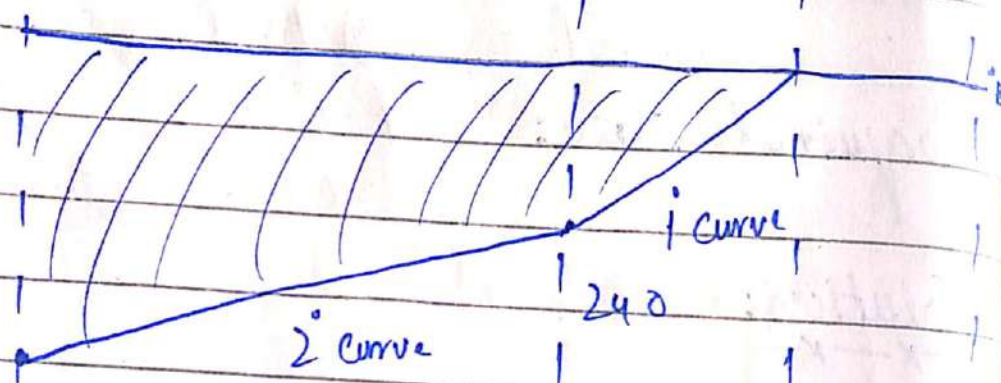
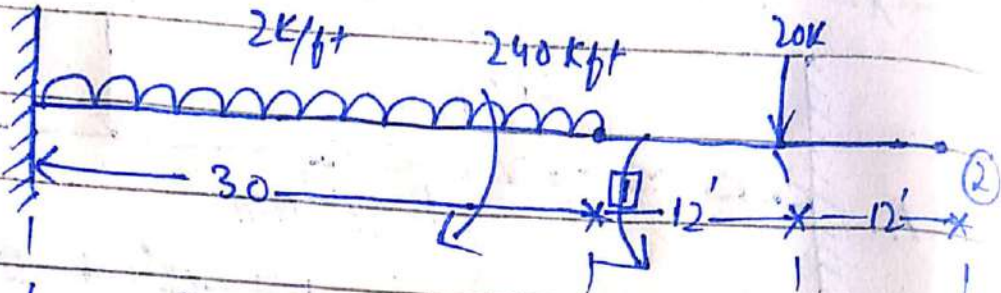


$$[AR] = \begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix}$$

$$[DRS] = [DRS_1] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

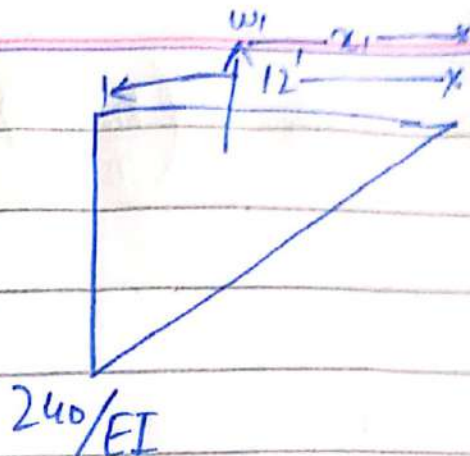


Compute [DRL] Matrix.



$$W_1 = \frac{1}{2} \times 12 \times \frac{240}{EI}$$

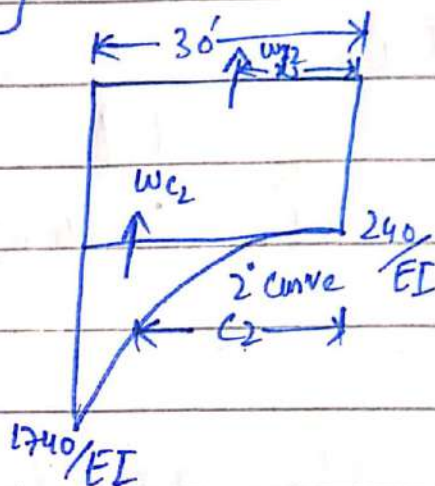
$$W_1 = \frac{1440}{EI} \text{ K}$$



$$x_1 = \frac{2}{3} (12) = 8 \text{ feet}$$

$$W_{x_2} = 30 \times \frac{240}{EI}$$

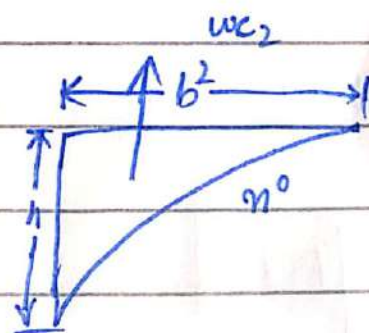
$$W_{x_2} = \frac{7200}{EI} \text{ K}$$



$$x_2 = \frac{1}{2} (30)$$

$$x_2 = 15 \text{ (ft)}$$

$$W_{C_2} = A = \frac{bh}{n+1}$$



$$W_{C_2} = \frac{30 \times 1500}{EI}$$

$$h = \frac{1740}{EI} - \frac{240}{EI}$$

$$W_{C_2} = \frac{15000}{EI} \text{ K}$$

$$h = \frac{1500}{EI}$$

$$C_2 = \left(\frac{n+1}{n+2} \right) (b) = \left(\frac{2+1}{2+2} \right) (30)$$

$$C_2 = 22.5 \text{ ft}$$

$$DRL_1 = w_1 x_1 + w_2 x_2$$

$$DRL_1 = \frac{7200}{EI} \times 15 + \frac{15000}{EI} \times 22.5$$

$$DRL_1 = 445500 / EI$$

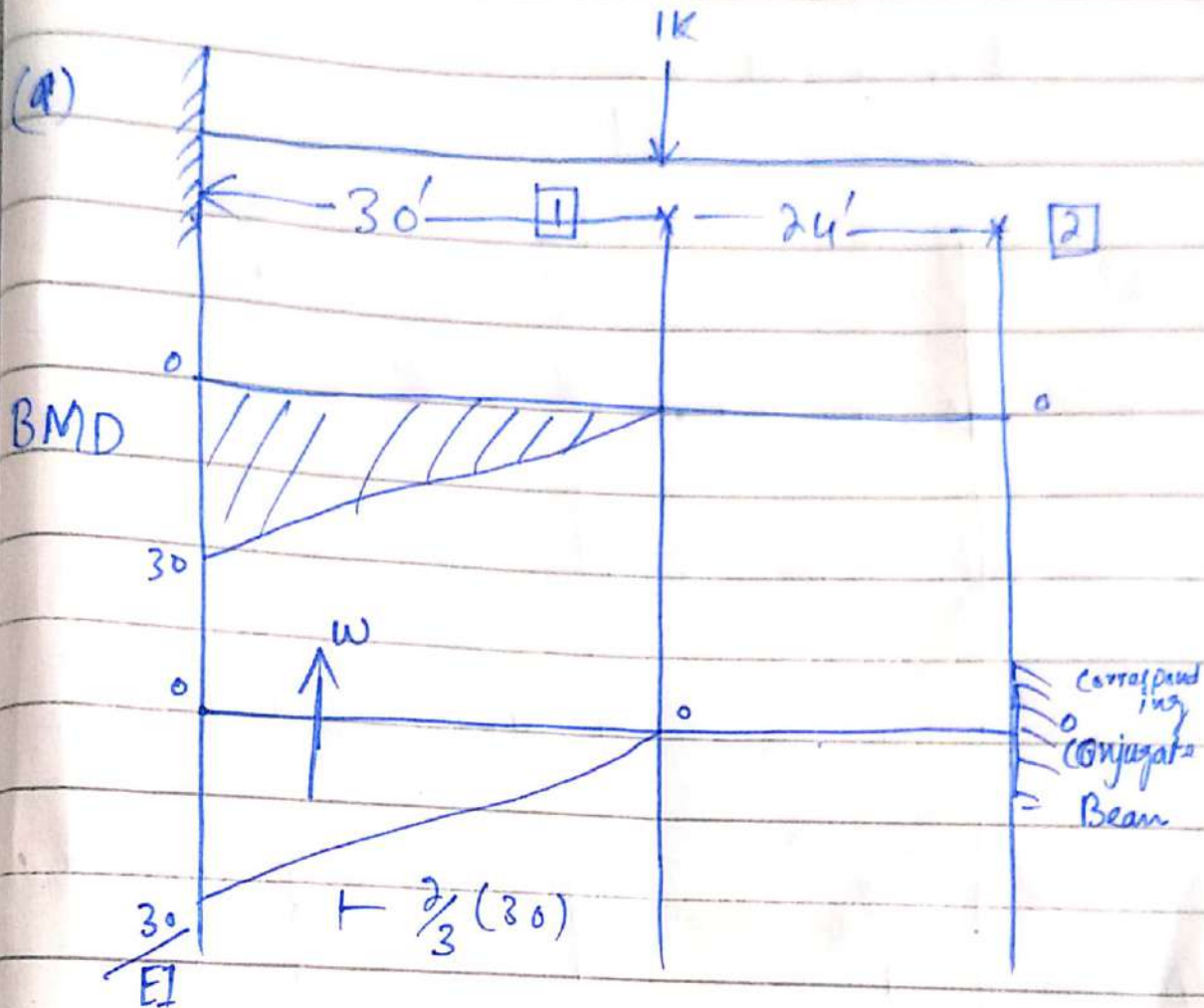
$$DRL_2 = w_1 (x_1 + 24) + w_2 (24 + C_2) + w_3 (8 + 12)$$

$$DRL_2 = \frac{7200}{EI} (15 + 24) + \frac{15000}{EI} (24 + 22.5) + \frac{1440}{EI} (8 + 12)$$

$$DRL_2 = 1007100 / EI$$

$$[DRL] = \begin{bmatrix} DRL_1 \\ DRL_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 445500 \\ 1007100 \end{bmatrix}$$

Compute flexibility matrix [F]



$$w = \frac{1}{2} \times 30 \times \frac{30}{EI}$$

$$w = \frac{450}{EI}$$

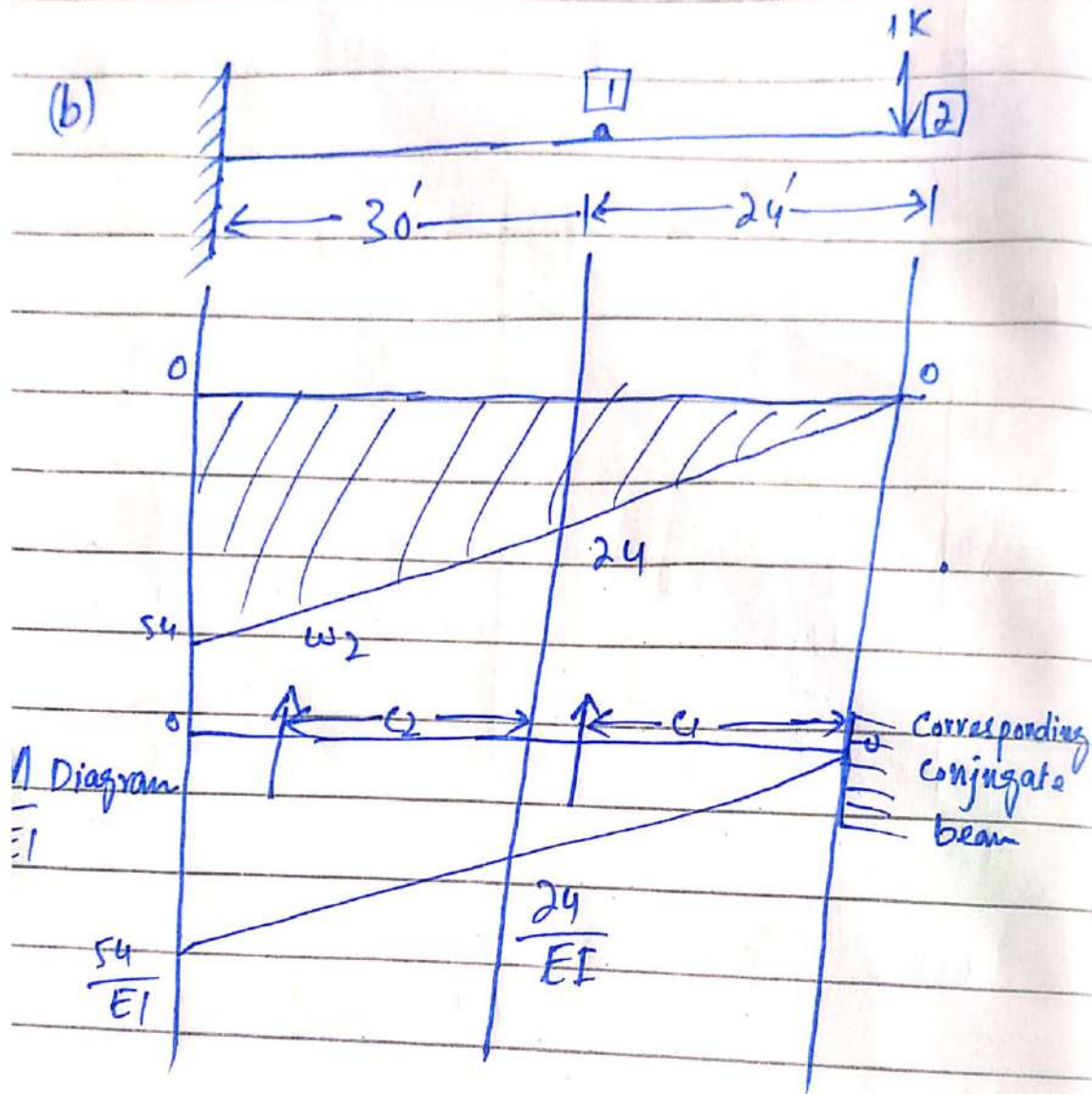
$$\text{Moment arm} = \frac{2}{3}(20) = 20$$

$$\Delta_{11} = F_{11} = \frac{450}{EI} \times 20$$

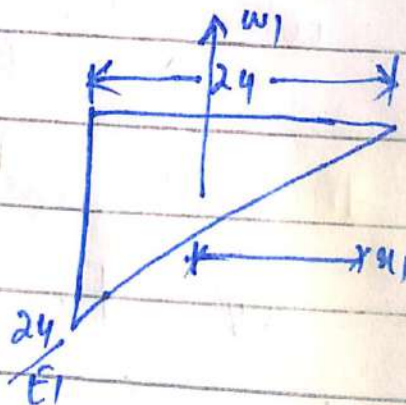
$$F_{11} = \frac{9000}{EI}$$

$$\Delta_{21} = F_{21} = \frac{410}{EI} \times (24 + 20)$$

$$F_{21} = 19800/EI$$

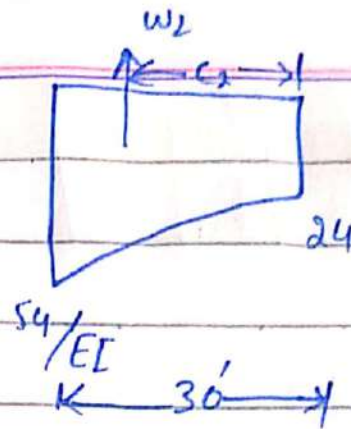


$$w_1 = \frac{1}{2} \times 24 \times \frac{24}{EI}$$



$$x_1 = \frac{2}{3} (24)$$

$$x_1 = 16 \text{ ft}$$



$$W_2 = \left(\frac{a+b}{2} \right) L = \left(\frac{54/EI + 24/EI}{2} \right) (30)$$

$$W_2 = 1170/EI$$

$$c_2 = \frac{L}{3} \left(\frac{2a+b}{a+b} \right) = \frac{30}{3} \left(\frac{2(54) + 24}{54 + 24} \right)$$

$$c_2 = 16.92 \text{ ft}$$

$$\Delta_{11} = f_{12} = W_2 \times c_2 = \frac{1170}{EI} \times 16.92$$

$$F_{12} = 19800/EI$$

$$\Delta_{22} = f_{22} = W_1 \times x_1 + W_2 \times (c_2 + 24)$$

$$f_{22} = 52485/EI$$

$$[f] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 9000 & 19800 \\ 19800 & 52485 \end{bmatrix}$$

Now

Apply the Compatibility equation

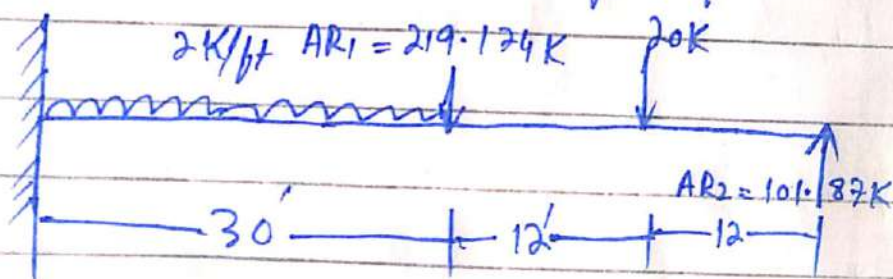
$$[DRS] = [DRL] + [F](AR)$$

$$[AR] = [f]^{-1} (DRS - DRL)$$

$$[AR] = \frac{1}{EI} \begin{bmatrix} 9000 & 19800 \\ 19800 & 52485 \end{bmatrix}^{-1} \begin{bmatrix} 0 - 445500 \\ 0 - 1007100 \end{bmatrix} \frac{1}{EI}$$

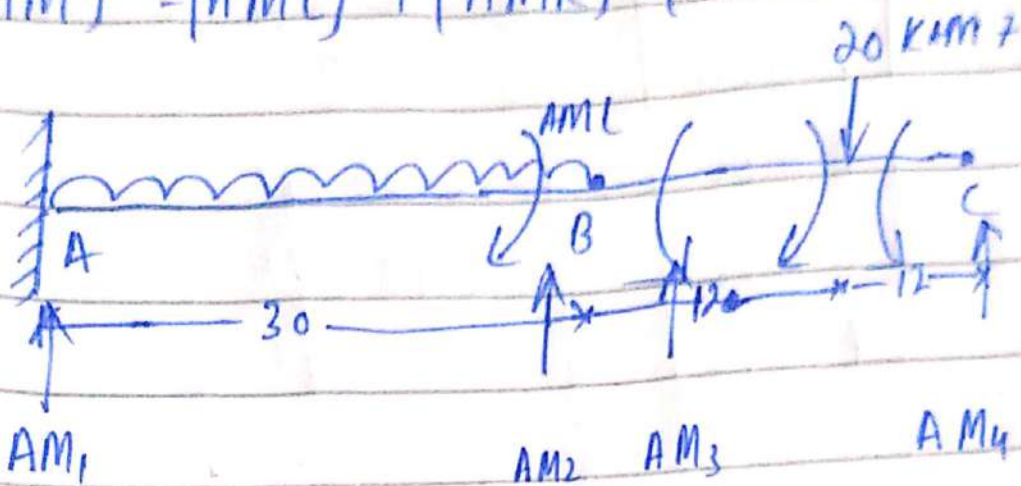
$$[AR] = \begin{bmatrix} 219.174 \text{ K} \\ -101.87 \text{ K} \end{bmatrix}$$

Where $AR_2 = -101.87 \text{ K}$ shows that the chosen direction is incorrect and thus acts vertically upward

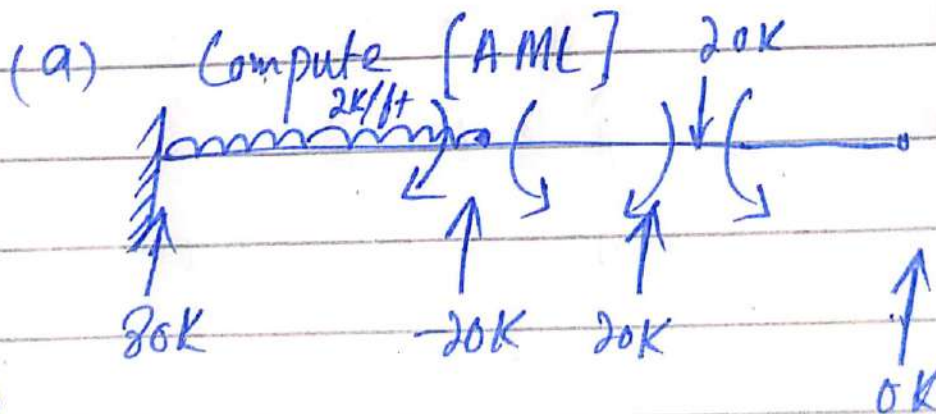


Compute members end actions

$$[AM] = [AML] + [AMR] [AR]$$

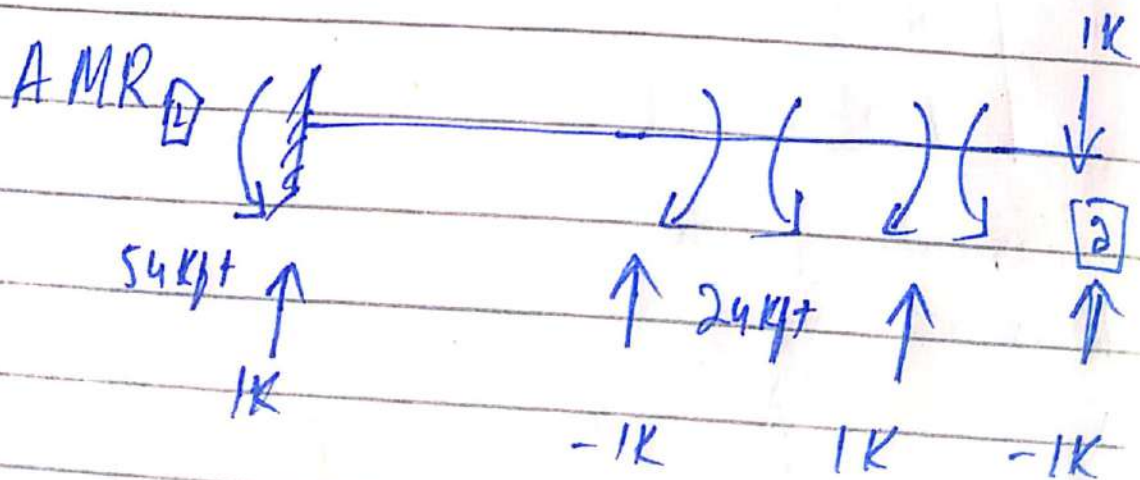
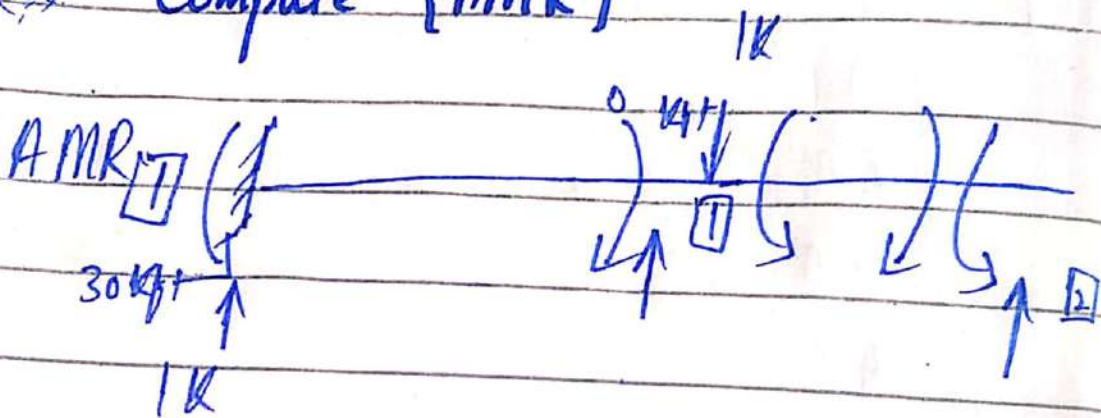


$$[AM] = \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$



$$[AML] = \begin{bmatrix} AML_1 \\ AML_2 \\ AML_3 \\ AML_4 \\ AML_5 \\ AML_6 \\ AML_7 \end{bmatrix} = \begin{bmatrix} 80 K \\ -20 K \\ 20 K \\ 0 K \\ 1740 Kft \\ 240 Kft \\ 0 Kft \end{bmatrix}$$

(b) Compute [AMR]

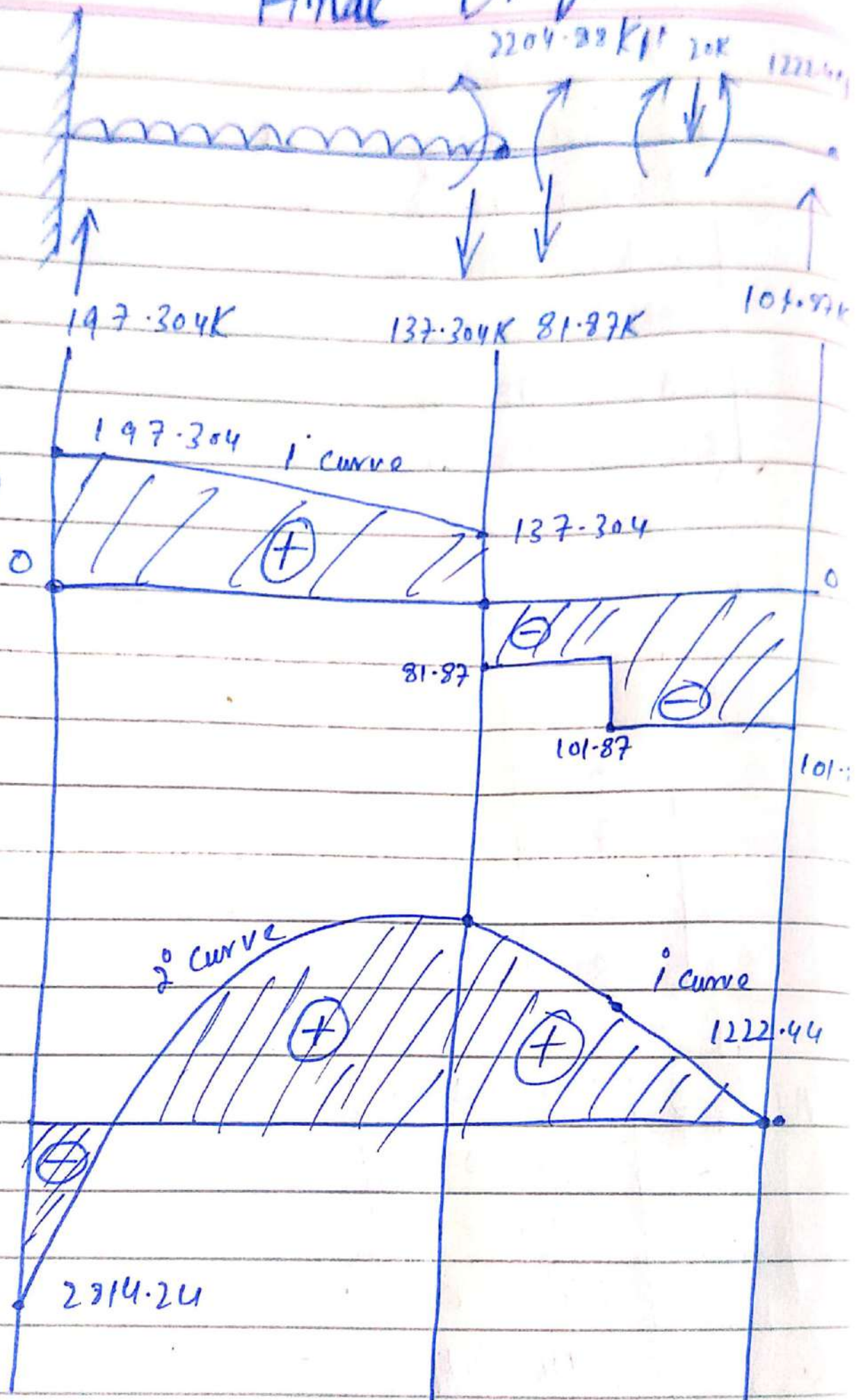


$$AMR = \begin{bmatrix} AMR_{11} & AMR_{12} \\ AMR_{21} & AMR_{22} \\ AMR_{31} & AMR_{32} \\ AMR_{41} & AMR_{42} \\ AMR_{51} & AMR_{52} \\ AMR_{61} & AMR_{62} \\ AMR_{71} & AMR_{72} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 30 & 54 \\ 0 & 24 \\ 0 & 12 \end{bmatrix}$$

$$[AM] = \begin{bmatrix} 80 \\ -20 \\ 20 \\ 0 \\ 1740 \\ 240 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ -1 & -1 \\ 0 & 1 \\ 0 & -1 \\ 30 & 54 \\ 0 & 24 \\ 0 & 12 \end{bmatrix} \begin{bmatrix} 219.174K \\ -101.87K \end{bmatrix}$$

$$[AM.] = \begin{bmatrix} 197.304K \\ -137.304K \\ -81.87K \\ 101.87K \\ 2814.24K \\ -2204.88K \\ -1222.44K \end{bmatrix}$$

Final Diagram



Q02 :-

Force Method

- (1) $D_s < D_k$
- (2) Forces are redundant or unknown
- (3) Starts with equilibrium of forces
- (4) Forces found by compatibility equations of displacements
- (5) No of Redundants = D_s
- (6) Not suitable for compression

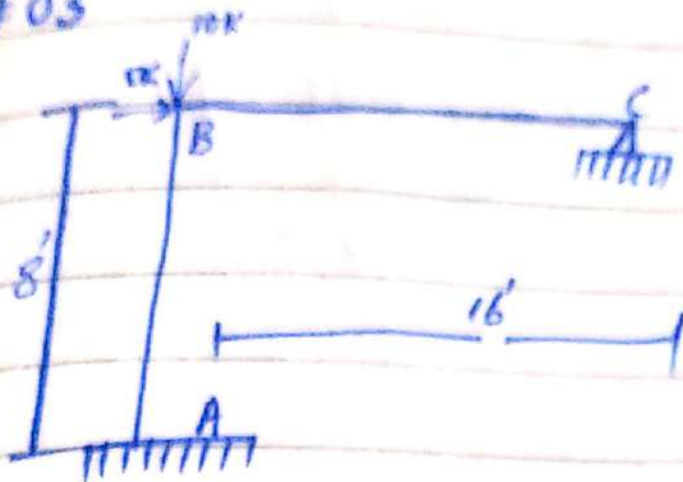
Displacement Method

- $D_s > D_k$
- Displacement are redundant or unknown
- Starts with compatible deformations
- displacement found by equilibrium equations of forces
- No of redundants = D_k
- Not suitable for trusses.

Stiffness method also called Displacement Method is more suitable for structure analysis matrix approach, as it is a primary method used in matrix analysis. The main advantage of this method over flexibility method

is that it is conducive to computer programming. Once the analytical model of the structure has been defined, no further engineering decisions are required in the stiffness method in order to carry out the analysis.

Pb # 03



$$E = \text{Constant}$$

$$I_C = I$$

$$I_B = 2I$$

Sol

Total Statical Indeterminacy
 $\Rightarrow R - 3 = 5 - 3 = 2$

Step # 01

Identify Redundant Action

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step 2 :-

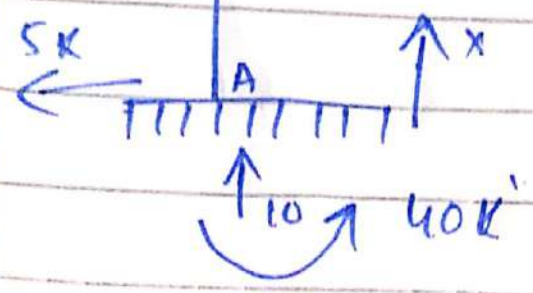
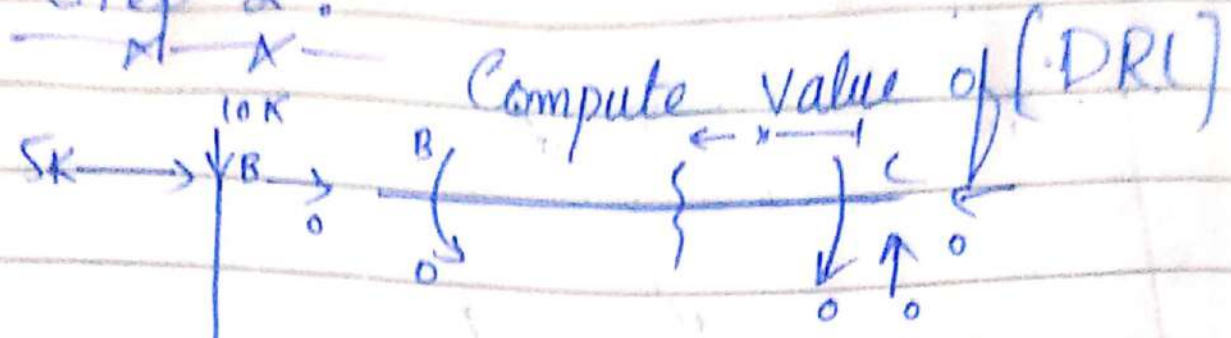


Fig: AML values (M-values)

Step No 3 :- [F] or [AMR]

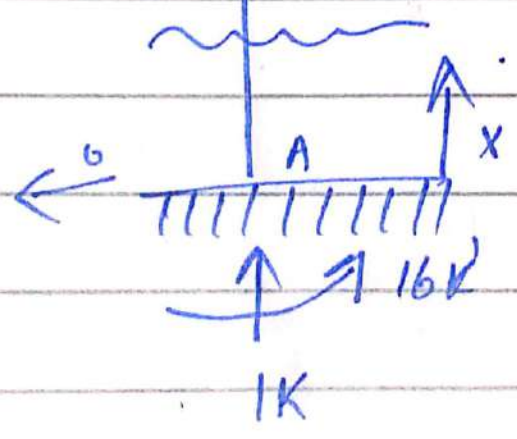
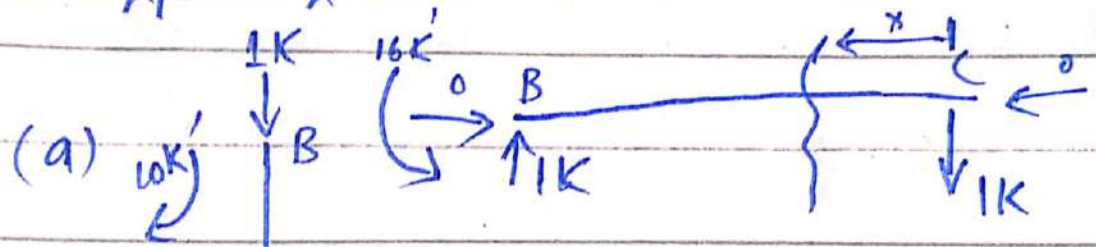


Fig AMR values (M₁ values)

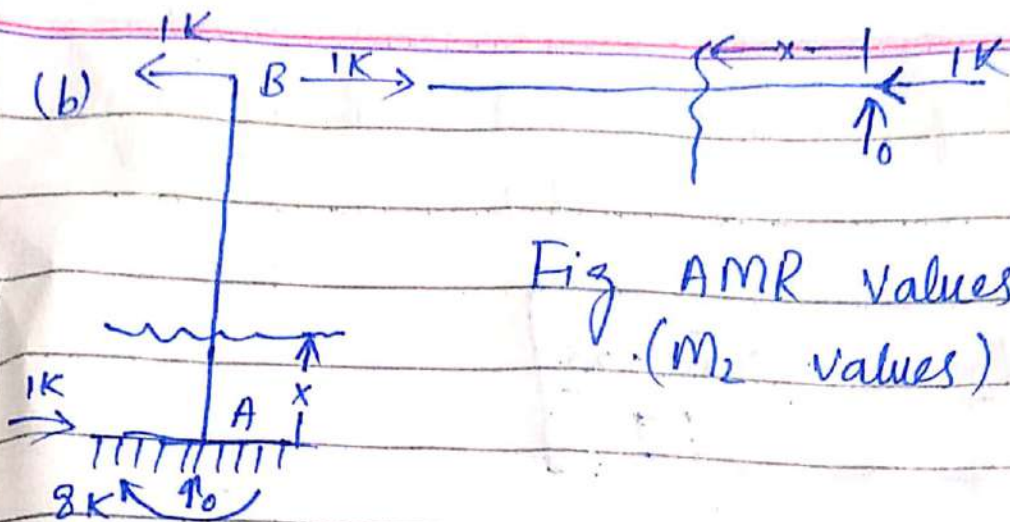


Fig AMR values
(M_2 values)

Member	AB	BC
origin	A	C
Limits	0-8	0-16
I	\bar{I}	$2\bar{I}$
M	$5x-40$	0
m_1	-16	$\odot x \rightarrow$
m_2	$8-x$	0

Take 'x' section on m_1 Fig from the origin

\Rightarrow For finding The value of DRL

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot m_1(AB)}{EI} dx + \int_0^{16} \frac{m_{BC} \cdot M_2(BC)}{E(2I)} dx$$

$$= \int_0^8 \frac{(5x-40)(-16)}{EI} dx + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx$$

$$DRL_1 = \frac{2560}{EI}$$

$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x)}{EI} dx + \int_0^{16} \frac{0 \cdot 0 \cdot d}{EI}$$

$$DRL_2 = \frac{-853.33}{EI}$$

⇒ Compute Flexibility Matrix :-

$$F_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$\begin{aligned} \Rightarrow F_{11} &= \int_0^8 \frac{m_1^2(AB)}{EI} + \int_0^{16} \frac{m_1(BC)}{EI} \\ &= \int_0^8 \frac{(-16)^2 dx}{EI} + \int_0^{16} \frac{x^2}{EI} dx \end{aligned}$$

$$F_{11} = \frac{2730.67}{EI}$$

$$F_{12} = F_{21} = \int_0^8 M_1(AB) \cdot M_2(AB) dx + \int_0^{16} m_2(BL) \cdot m_2(BC) dx$$

$$= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{2EI} dx$$

$$F_{12} = F_{21} = -\frac{512}{EI}$$

$$F_{22} = \int_0^8 (M_2)^2_{AB} dx + \int_0^{16} (m_2)^2_{BL} dx$$

$$= \int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{2EI} dx$$

$$F_{22} = 170.67$$

As we know that

$$[DRS] = [DRL] + [AR] \times [F]$$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$(2) [AR] = [F]^{-1} \times [DRS - DRL]$$

$$= \begin{bmatrix} 2730 \cdot 67 - 1512 \\ -512 \\ 170 \cdot 67 \end{bmatrix} \times \begin{bmatrix} 0 - 2560 \\ 0 + 853 \cdot 33 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.00005 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

