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Section "B"

Subject: D.E

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(1)

### Question No#01

A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian, and American Cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kilogram of A, B, C costs 40, 50 and 60 Rupees respectively. Find the cost of a kilogram of cotton of each country.

Solution.

40

P	E
A	E

B<sub>1</sub>

50

P	P
A	E

B<sub>2</sub>

60

P	P
A	A

B<sub>3</sub>

Let  $x, y, z$  be the cost of cost of Pakistani, Egyptian and American kg Cotton respectively. Then according to the given conditions.



(2)

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \end{aligned} \right\} \dots (S')$$
$$\frac{2}{4}x + \frac{2}{4}z = 60$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 1600 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} \text{---} (S)$$

In matrix form, we can write as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 1600 \\ 200 \\ 120 \end{bmatrix}$$



$$A\underline{x} = \underline{b}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \underline{b} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$



$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1 \times 1 - 0 \times 1) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 1 - 1 \times 1)$$

$$|A| = -2,$$

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} = 1(1 \times 1 - 0 \times 1) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 1 - 1 \times 1)$$

$$|A_1| = -2,$$

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix} = 160(1 \times 1 - 0 \times 1) - 2(200 \times 1 - 120 \times 1) + 1(200 \times 1 - 120 \times 1)$$

$$|A_1| = -120,$$

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} = 1(200 \times 1 - 120 \times 1) - 160(2 \times 1 \times 1) + 1(2 \times 1 - 1 \times 200)$$

$$|A_2| = -40$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} = 1(1 \times 120 - 0 \times 200) - 2(2 \times 120 - 1 \times 200) + 160(2 \times 120 - 1 \times 1)$$

$$|A_3| = +120,$$



(5)

Hence  $|A| = -2$

$$|A_1| = -20$$

$$|A_2| = -40$$

$$|A_3| = -120$$

According to Cramers Rule.

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

So Pakistani = Rs. 60 per kg.

Egyptian = Rs. 20 per kg.

American = Rs. 60 per kg.



To check the Ans.

Such as  $(x, y, z) = (60, 20, 60)$

So this value put in 3 equations.

$$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40 \quad \text{--- (1)}$$

Put  $x = 60$   $y = 20$   $z = 60$  in eq (1)

$$\frac{1}{4}(60) + \frac{2}{4}(20) + \frac{1}{4}(60) = 40$$

$$40 = 40$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50 \quad \text{--- (2)}$$

Put  $x = 60$ ,  $y = 20$ ,  $z = 60$  in eq (2)

$$\frac{2}{4}(60) + \frac{1}{4}(20) + \frac{1}{4}(60) = 50$$

$$\boxed{50 = 50}$$

$$\frac{2}{4}x + \frac{2}{4}z = 60$$

Put  $x = 60$ ,  $y = 20$   $z = 60$  in eq (3)

$$\frac{2}{4}(60) + \frac{2}{4}(60) = 60$$

$$\boxed{60 = 60}$$

left hand side is equal to the Right hand side of Three equation is correct.