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Sec A

Subject Probability & statistics

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Q 1.

Compute and analyse the result of the least squares regression equation and coefficient of correlation of Y on X for the following data. Compare your manual results with the outcome of SPSS?

X =	Temperature	53	62	57	71	78	64	86	87	96	91	94	94
Y =	Chirps per minute	20	32	45	60	80	100	120	140	160	180	200	210

The estimated regression lines of Y on X is

$$\hat{Y} = a + bx$$

and two normal equations are

$$\sum Y = a + b \sum X$$

$$\sum XY = a \sum X + b \sum X^2$$

To compute the necessary summation, we arrange the computations in the table below.

X	Y	XY	X ²	Y ²
53	20	1060	2809	400
62	32	1984	3844	1024
57	45	2565	3249	2025
71	60	4260	2601	3600
78	80	6240	6084	6400
64	100	6400	4096	10,000
86	120	10320	7396	14400
87	140	12180	7469	19600
96	160	15360	9210	25600
91	180	16380	8281	32400
94	200	18800	8836	40000
94	210	19740	8836	44100
933	1347	115,289	72,711	199,549

Now

$$\bar{x} = \frac{\sum x}{n} = \frac{933}{12} = 77.75$$

$$\bar{y} = \frac{\sum y}{n} = \frac{1347}{12} = 112.25$$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{(12)(115,289) - (933)(1347)}{(12)(72,711) - (933)^2}$$

$$b = \frac{1383468 - 1256751}{872532 - 870489}$$

$$b = \frac{126717}{2043}$$

$$b = 62.02 \quad \text{and}$$

$$a = \bar{Y} - b\bar{X}$$

$$a = 77.75 - 62.02(112.25)$$

$$a = -6883.9$$

Hence the desired estimated regression line of Y on X is

$$\hat{Y} = (a + bx) = (-6883.9) + 62.02X$$

$$\hat{Y} = 62.02X - 6883.9$$

~~Hence the desired estimated regression line of Y on X is~~

Hence $\hat{Y} = 62.02X - 6883.9$ is the estimated regression equation for predicting the Y , given the X .

b)

$$r = \frac{\sum XY - (\sum X)(\sum Y)/n}{\sqrt{[\sum X^2 - (\sum X)^2/n][\sum Y^2 - (\sum Y)^2/n]}}$$

$$= \frac{115289 - (933)(1347)/12}{\sqrt{[72711 - (933)^2/12][199549 - (1347)^2/12]}}$$

$$= \frac{10,559.75}{\sqrt{(170.25)(48348.25)}}$$

$$= \frac{10,559.75}{2,869.02}$$

$$r = 3.68$$

Coefficient of correlation.

Q2.(a)

A box contain 4 red, 4 white and 5 green balls are drawn from the box together. Find the probability that they may be (i) all of different colours
ii) all of the same colour.

Solution:-

The total number of possible equally outcomes is S ;

$$n(S) = \binom{13}{3}$$

$$n(S) = \frac{13!}{3!(13-3)!}$$

$$n(S) = 286$$

i) Let X represent "All of different colour"

$$n(X) = \binom{4}{1} \binom{4}{1} \binom{5}{1}$$

$$n(X) = \left(\frac{4!}{1!(4-1)!} \right) \left(\frac{4!}{1!(4-1)!} \right) \left(\frac{5!}{1!(5-1)!} \right)$$

$$n(X) = (4)(4)(5)$$

$$n(X) = 80$$

So the probability is;

$$P(x) = \frac{n(x)}{n(S)}$$

$$P(x) = \frac{80}{286}$$

$$P(x) = 0.28$$

ii) Let Y represents "All of same colour"

$$n(Y) = \binom{4}{3} + \binom{4}{3} + \binom{5}{3}$$

$$n(Y) = \frac{4!}{3!(4-3)!} + \frac{4!}{3!(4-3)!} + \frac{5!}{3!(5-3)!}$$

$$n(Y) = 4 + 4 + 10$$

$$n(Y) = 18$$

So the probability is;

$$P(Y) = \frac{n(Y)}{n(S)}$$

$$P(Y) = \frac{18}{286}$$

$$P(Y) = 0.063$$

Q 2. (b)

Of 12 eggs in a refrigerator 2 are bad. From these, 4 eggs are chosen at random to make a cake. What are the probabilities that (i) exactly one is bad?

Solution:-

The total number of possible equally likely outcomes is S ,

$$n(S) = \binom{12}{4}$$

$$n(S) = \frac{12!}{4!(12-4)!} = 495$$

(i) Let X represent "Exactly one is bad"

$$n(X) = \binom{2}{1} \binom{10}{3}$$

$$n(X) = \left(\frac{2!}{2!(2-1)!} \right) \left(\frac{10!}{3!(10-3)!} \right)$$

$$n(X) = (2)(120)$$

$$n(X) = 240$$

So the probability is

$$P(x) = \frac{n(x)}{n(S)} = \frac{240}{495}$$

$$P(x) = 0.48$$

ii) Let Y represent (At least one is bad)

$$n(Y) = \binom{10}{3} \binom{2}{1} + \binom{10}{2} \binom{2}{2}$$

$$n(Y) = \frac{10!}{3!(10-3)!} \left(\frac{2!}{1!(2-1)!} \right) + \left(\frac{10!}{2!(10-2)!} \right) \left(\frac{2!}{2!(2-2)!} \right)$$

$$n(Y) = (120)(2) + (45)(1)$$

$$n(Y) = 240 + 45$$

$$n(Y) = 285$$

So the probability is

$$P(Y) = \frac{n(Y)}{n(S)}$$

$$P(Y) = \frac{285}{495}$$

$$P(Y) = 0.58$$

Q3.

Following are the scores made by batsman A, B and C.

A	12	15	6	73	7	64	199	36	84	29
B	47	12	76	48	4	64	37	48	13	34
C	15	23	52	4	24	64	74	52	13	4

Find the range of A, B and C who is more consistent. Compare A with B, B with C and A with C.

Solution:-

A	B	C
12	47	15
15	12	23
6	76	52
73	48	4
7	4	24
64	64	64
199	37	74
36	48	52
84	13	13
29	3	4

$$\text{Range} = X_m - X_0$$

$$\text{Range of A} = X_m - X_0$$

$$= 199 - 6$$

$$= 193$$

$$= 193$$

$$\text{Range of B} = X_m - X_0$$

$$= 76 - 3$$

$$= 73$$

$$= 73$$

$$\text{Range of C} = X_m - X_0$$

$$= 74 - 4$$

$$= 70$$

$$= 70$$

Batsman A		Batsman B		Batsman C	
X	X ²	Y	Y ²	Z	Z ²
12	144	47	2209	15	225
15	225	12	144	23	529
6	36	76	5776	52	2704
73	5329	48	2304	4	16
7	49	4	16	24	576
64	4096	64	4096	64	4096
199	39601	37	1369	74	5476
36	1296	48	2304	52	2704
84	7056	13	169	13	169
29	841	3	9	4	16
$\Sigma X = 525$	$\Sigma X^2 = 84,571$	$\Sigma Y = 352$	$\Sigma Y^2 = 73,369$	$\Sigma Z = 325$	$\Sigma Z^2 = 16,511$
	58,673				

Batsman A.

$$\bar{x} = \frac{\sum x}{n} = \quad \because n=10$$

$$= \frac{525}{10} = 52.5$$

$$S_x = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$= \sqrt{\frac{58673}{10} - \left(\frac{525}{10}\right)^2}$$

$$S_{\bar{x}} = 10.53$$

$$C.V = \frac{10.53 \times 100}{\bar{x}} = \frac{10.53}{52.5}$$

$$C.V = 0.20 \times 100$$

$$C.V = 20.05$$

Batsman B

$$y = \frac{\sum y}{n} = \frac{352}{10} = 35.2$$

$$S_y = \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2}$$

$$S_y = \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2}$$

$$S_y = \sqrt{\frac{73369}{10} - \left(\frac{352}{10}\right)^2}$$

$$S_y = \sqrt{7336.9 - 1239.04}$$

$$S_y = 6097.86$$

$$C.V = \frac{6097.86 \times 100}{35.2}$$

$$C.V = 173.2 \times 100$$

$$C.V = 173.2\%$$

Batsman C.

$$Z = \frac{\sum z}{n} = \frac{325}{10} = 32.5$$

$$S_z = \sqrt{\frac{\sum z^2}{n} - \left(\frac{\sum z}{n}\right)^2}$$

$$S_z = \sqrt{\frac{16511}{10} - \left(\frac{325}{10}\right)^2}$$

$$S_z = 594.85$$

$$C.V = \frac{S^2}{\bar{X}}$$

$$= \frac{594.85}{32.5}$$

$$= 18.30$$

Batsman B is more consistent as its value of coefficient of variance is smallest

Compare A with B

B is consistent

Compare B with A

B is more consistent

Compare A with C

C is more consistent