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Section # B

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Subject # calculus

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Q1

FIND PQ where P is the point in three-dimensional space with coordinates (4,1,3) and the point Q with coordinate (1,2,4). FIND the Distance b/w P & Q Further, Find the position vector of the point dividing PQ in the ratio 1:3.

Solution :-

coordinate of P = (4,1,3)

$OP = 4i + 1j + 3k$

or $OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k \longrightarrow (1)$

Now distance between P & Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11} \longrightarrow (2)$

Let M be The point which divided PQ in ratio 1:3 Then by The ratio theorm position vector of $M = \vec{OM}$

$$= \frac{3(4i+1j+3k)+(1)(i+2j+4k)}{1+3}$$

$$= \frac{12i+3j+9k+i+2j+4k}{4}$$

$$= \frac{13i+5j+13k}{4} \longrightarrow (3)$$

Hence eq 1, 2 & 3 are required Solution.

Q No # 2

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} \, dx$$

Solution :-

$$\begin{array}{r|l} & 2x - 1 \\ \hline 2x^2+x & \cancel{4x^3} + 10x + 4 \\ & \pm \cancel{4x^3} \pm 2x^2 \\ \hline & -\cancel{2x^2} + 10x + 4 \\ & \pm \cancel{2x^2} \mp x \\ \hline & 11x + 4 \end{array}$$

So $2x - 1 + \dfrac{11x + 4}{2x^2 + x} = \dfrac{4x^3 + 10x + 4}{2x^2 + x}$

$$\Rightarrow \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \longrightarrow (1)$$

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(4)

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow (2)$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow (A)$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1) + Bx \longrightarrow (3)$$

put $x=0$ in (3)

$$\boxed{4 = A}$$

Now put $x = -1/2$ in (3)

$\Rightarrow 11(-1/2) + 4 = B(-1/2)$

$\Rightarrow -11/2 + 4 = -B/2$

$\Rightarrow -11 + 8/\cancel{2} = -B/\cancel{2}$

$\Rightarrow -3 = -B \Rightarrow \boxed{B = 3}$

putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = 4/x + 3/2x+1$$

Taking integral on both side

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

putting The value in (2)

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1)$$

Now put These values in (1)

$$\boxed{\frac{4x^3 + 10x + 4}{2x^x + x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C}$$

Ans

Q3 part a:

$$\int_0^2 x^2 e^x \, dx$$

Solution :-

$$= x^2 e^x \Big|_0^2 - \int_0^2 e^x (2x) \, dx$$

$$= x^2 e^x \Big|_0^2 - 2 \left\{ \int_0^2 e^x x \, dx \right\}$$

$$= x^2 e^x \Big|_0^2 - 2 \left\{ x e^x \Big|_0^2 - \int_0^2 e^x (1) \, dx \right\}$$

$$= x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2 + 2 \int_0^2 e^x \, dx$$

$$= x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2 + 2e^x \Big|_0^2$$

$$= (2)^2 e^2 - (0) - \left\{ 2(2) e^2 - 0 \right\} + 2e^2 - 2e^0$$

$$= 2e^2 - 2$$

Ans

$$= 12.77$$

Q3 part b:

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Solution:-

Let $u = \sqrt{x}$ so $du = \sqrt{x}/2x \; dx$

Limits at $x = 1$ at $x = 2$. $u = \sqrt{2}$

The original equation in variable u becomes.

$$= \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_1^{\sqrt{2}} \frac{\sin u}{u} \cdot 2u\,dx$$

$$= 2\int_1^{\sqrt{2}} \frac{\cancel{u} \cdot \sin u\,du}{\cancel{u}}$$

$$= 2\int_1^{\sqrt{2}} \sin u\,du$$

$$= 2\left(-\cos u \Big|_{1}^{\sqrt{2}}\right)$$

$$= -2\left(\cos\sqrt{2} - \cos\phi\right)$$

$$= 2\cos(1) - 2\cos(\sqrt{2})$$

Ans :-

Q4 Verify That :

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

The Laplace equation in 3. d is

$$\partial^2 u / \partial x^2 + \frac{\partial^2 u}{\partial y^2} + \partial^2 u / \partial z^2 = 0 \longrightarrow (A)$$

so $u(x, y, z) = 1 / \sqrt{x^2 + y^2 + z^2}$

$$u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -1/\cancel{2} (x^2 + y^2 + z^2)^{-3/2} (\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2 + y^2 + z^2)^{-3/2}$$

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$$= \frac{\partial^2 u}{\partial x^2} = -\left[x\left(-3/\cancel{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}\left(\cancel{2}x\right) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= \frac{\partial^2 u}{\partial x^2} = 3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} \quad (1)$$

Now:-

$$\frac{\partial u}{\partial y} = -1/\cancel{2}\left(x^2+y^2+z^2\right)^{-3/2}\left(\cancel{2}y\right)$$

$$= \frac{\partial u}{\partial y} = y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial y^2} = -\left[y\left(-3/\cancel{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}\left(\cancel{2}y\right) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= \boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}} \rightarrow (2)$$

$$= \frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$= \frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$= \boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)]]^{-3/2}} \rightarrow (3)$$

putting value of eq (1) (2) & (3) in

(A)

$$\Rightarrow 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$\Rightarrow (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2) + 3z^2-(x^2+y^2+z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3\cancel{x^2}-\cancel{x^2}-\cancel{y^2}+z^2+3y^2+x^2-\cancel{y^2}-z^2+3z^2+x^2-\cancel{y^2}-z^2\right]$$

$$= \left(x^2+y^2+z^2\right)^{-5/2} \left(0\right] = 0$$

So This given $u(x,y,z)$ is solution of laplace equation.