Subject: Calculus

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QUESTION No. 1

Find PQ where P is the point in 3D with coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance b/w P and Q. Further, find the position vector of the point dividing PQ in the ratio ~~of~~ 1:3.

SOLUTION

$$P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$$
$$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$$

Now distance b/w PQ

So, $|PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$$= \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$$
$$= \sqrt{3^2 + (-1)^2 + (-1)^2}$$
$$|PQ| = \sqrt{9+1+1} = \sqrt{11}$$

Now find position vector of the point dividing PQ in the ratio of 1:3

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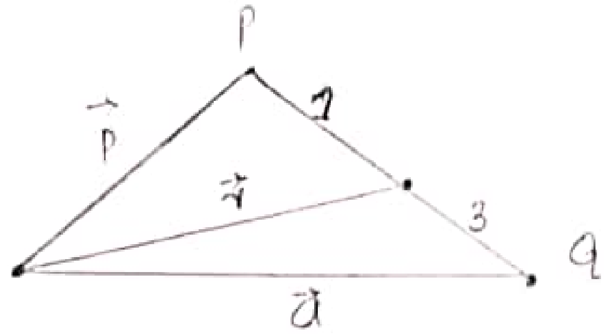

$a : b = 1 : 3$

$$\vec{r} = \frac{b\vec{p} + a\vec{a}}{b + a}$$

$$= \frac{3(4\hat{i} + \hat{j} + 3\hat{k}) + 1(\hat{i} + 2\hat{j} + 4\hat{k})}{3 + 1}$$

$$= \frac{12\hat{i} + 3\hat{j} + 9\hat{k} + \hat{i} + 2\hat{j} + 4\hat{k}}{4}$$

$$= \frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}}$$

Question No. 2

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

Solution

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

$$\begin{array}{r|l}
 & 2x-1 \\
\hline
2x^3+x & 4x^3+10x+4 \\
 & \underline{\pm 4x^3 \qquad\qquad +2x^2} \\
 & -2x^2+10x+4 \\
 & \underline{\ominus 2x^2 \ominus x} \\
 & +\qquad\ \ + \\
 & 11x+4
\end{array}$$

So,

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}$$

$$\int \frac{4x^3-10x+4}{2x^2+x}=\int 2x-1+\frac{11x+4}{2x^2+x} \quad \text{——(1)}$$

$$=2\int x\,dx-\int 1\,dx+\int \frac{11x+4}{2x^2+x}\,dx$$

$$=\frac{2x^2}{2}-x+\int \frac{11x+4}{x(2x+1)}\,dx \quad \text{——(2)}$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = A/x + B/(2x+1) \longrightarrow (A)$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1)+Bx \longrightarrow (3)$$

Put $x=0$ in eq(3)

$$4 = A \text{ or } \boxed{A=4}$$

Now put $x = -1/2$ in eq(3)

$$11(-1/2)+4 = B(-1/2)$$

$$-11/2 + 4 = -B/2$$

$$-3 = -B \text{ or } \boxed{B=3}$$

Put the value of A and B in (A)

$$\frac{11x+4}{x(2x+1)} = 4/x + 3/(2x+1)$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)} = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Putting these values in eq (2)

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Now put these in 1

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x - 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Question No. 3

Evaluate

a. $\int_0^2 x^2 e^x \, dx$

Solution

First find integration

$$\int_0^2 x^2 e^x \, dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \; \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \; \frac{dx}{dx} \right) dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

$$= x^2 e^x - 2x e^x + 2e^x$$

Now put limits

$$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$$

$$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$$

$$= \left(4e^2 - 4e^2 + 2e^2 - 2 \right)$$

$$= \boxed{2e^2 - 2} \text{ Ans.}$$

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(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

Solution

First find integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \quad \text{---(1)}$$

Let $y = \sqrt{x}$

$$dy/dx = \frac{1}{2\sqrt{x}}$$

$$\boxed{2dy = \frac{1}{\sqrt{x}}\, dx} \Rightarrow \text{Put in eq(1)}$$

$$\int \sin(y)\,(2dy) = 2\int \sin(y)dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put y in above equation i.e. $y = \sqrt{x}$

Put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$$

$$= \boxed{-2\cos\sqrt{2} + 2\cos(1)} \text{ Ans.}$$

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Question No. 4

Verify that

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Stasfies the 3D- Laplace's equation

Solution

The laplace equation in 3D is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{--- (A)}$$

So, $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \quad \text{--- (1)}$$

Now

$$\frac{\partial u}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-3/2\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/5} - (x^2+y^2+z^2)^{-1/2} \quad \text{--- (2)}$$

$$\frac{\partial u}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} \quad \text{--- (3)}$$

Put (1) (2) and (3) in eq (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} +$$

$$3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}+\cancel{3y^2}-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}+\cancel{3z^2}-\cancel{x^2}-\cancel{y^2}-\cancel{z^2}\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution Laplace equation.

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