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Assignment - Linear Algebra
Department - BS(SE)2

Q NO 1:
consider the given below matrix as the augmented Matrix of a linear system. explain in your own word the next elementry row opreation that should be performed in order to Solve the system. where ID3 is the third Digit in your ID and ID Last is the Last digit in your ID. e.g of your ID.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\ Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Sol

$$\begin{bmatrix} 1 & 8 & 3 & 0 & 5 \\ 0 & 1 & -7 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

In linear form

$$x_1 + 8x_2 + 3x_3 + 0x_4 = 5$$
$$0x_1 + x_2 - 7x_3 + 0x_4 = 7$$
$$0x_1 + 0x_2 + x_3 + 0x_4 = -6$$
$$0x_1 + 0x_2 + 0x_3 + x_4 = 8$$

P.T.O

in Augmented form

$$\left[\begin{array}{cccc|c} 1 & 8 & 3 & 0 & 5 \\ 0 & 1 & -7 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right] \quad \begin{array}{l} R_2 \to 7R_3 + R_2 \\ R_1 \to 8R_2 - R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & -5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right] \quad R_1 \to 3R_3 + R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right]$$

$$x_1 + 0x_2 + 0x_3 + 0x_4 = -5$$

$$\boxed{x_1 = -5}$$

$$0x_1 + x_2 + 0x_3 + 0x_4 = 7$$

$$\boxed{x_2 = 7}$$

$$0x_1 + 0x_2 + x_3 + 0x_4 = -6$$

$$\boxed{x_3 = -6}$$

$$0x_1 + 0x_2 + 0x_3 + x_4 = 8$$

$$\boxed{x_4 = 8}$$

Solution = $\boxed{\{-5, 7, -6, 8\}}$

p.t.o

Q No 2:

part a:

Find the elementry row opreation that transfer the first matrix into second and reverse row opreation that transfer the second Matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

sol,

first Matrix

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \quad R_3 \to (-2)R_2 + R_3$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$ into second Matrix

Now reverse row opreation:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

P.T.O

(4)

(2) 0 1 -4 2
+0 0 3 -5
0 2 -5 -1

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix} \quad R_3 \rightarrow 2R_2 + R_3$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Q No 2

part B:

Given below are some Matrix find where these are the form written in the front of them or not Explain in your own word of each of the section in detail

(a)

$$\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

This is an echelon form beacuse it satisfy the conditio Ay row of all zero are below any other row. Each leading of a row is in a coloumn to the right of leading entry above it All entries in a column below a leading entries are zero.

P.T.O

(b)
$$\begin{bmatrix} 1 & 0 & \Omega \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This is an reduce echelon form of Matrix because it satisfy all the condition of the reduce echelon form. The leading entry of each row is (1). each leading (1) is the only non zero entry in its column.

(c)
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

This is not echelon form and not reduce echlon because it doesnot satisfy for both the condition of echelon and non reduce echelon form. All entries in a column below a leading entry are zero or any row of all zeros are below any other row.

(d)
$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

This is not echelon or not reduce echlon form of matrix because it doest not stasfy the condit

Q no 3:

part a:

(a) The row echelon form is used to solve the system of linear equation what is the difference between in row echelon and reduced row echelon form. Give one example of reduce echelon form.

Ans:

Row Echelon form:

In linear algebra a matrix is in echleon form if it has the shape resulting form a Gaussian elimanition

A matrix being in row echleon form mean that the Gaussian elimanition has opreated on the row

- all non zero row (rows with atleast one non zero element) are above any row of all zeros (all zeros rows) being at the bottom of the matrix
- The leading cofficient (the first non zero number from from the left also called the pivot) of the non zero row is always strictley to the right of the leading cofficient of the row

p.t.o

The following is an example of 3x5 matrix in row echelon form.

$$\begin{vmatrix} 1 & a_0 & a_1 & a_2 & a_3 \\ 0 & 0 & 2 & a_4 & a_5 \\ 0 & 0 & 0 & 1 & a_6 \end{vmatrix}$$

(ii) **Reduced row echelon form:-**

A matrix is in reduced row (also called row conical form) if it satisfies the folowing condition.

- The leading entry in each nonzero row is a 1 (called a leading 1)
- Each column contains a leading 1 has zeros in all other entries.

Example:

$$\begin{vmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{vmatrix}$$

This is an example of a matrix in reduced row echelon form which shows that the left part is not always in Identity matrix.

Q NO 3

part B:

P.T.O

(b) find an echelon form of the below matrix using row opreation where ID2 is the 2nd digit in your ID eg if your ID 12345 ID2 = 2, ID3 = 3 your ID first and last = 15

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID \text{ first} \end{bmatrix}$$

ID = 15837

sol,

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -8 & 0 & 0 \\ 1 & -4 & 17 \end{bmatrix}$$

Performing row opreation

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -8 & 0 & 0 \\ 1 & -4 & 17 \end{bmatrix}$$ Replace R_4 by R_3 $R_3 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 0 & 8 & -34 \\ -8 & 0 & 0 \end{bmatrix}$$ $R_3 \to R_2 - R_3$

P.T.O

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & -33 \\ -8 & 0 & 0 \end{bmatrix} \quad R_4 \rightarrow R_4 + 4R_2$$

$$\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ 0 & 0 & -33 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \rightarrow R_2 - 2R_1$$

$$= \begin{bmatrix} 1 & 5 & 8 \\ 0 & 8 & -1 \\ 0 & 0 & -33 \\ 0 & 0 & 0 \end{bmatrix}$$

Required echelon form.

END.