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Section A

Subject Applied calculus

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Question No 1

The function $g(t)$ is defined by $g(t) = 0$ $\quad t < 0$

$g(t) = 0$

t^2 ,, $\quad 0 \leq t \leq 3$

$2t+3 \quad 3 < t \leq 4$

$12 \quad t > 4$

(a) state any point of discontinuity

(b) Find, if they exist

i) $\lim_{t \to 3} g$

Sol:

ANSWER:- To check posibility of the discontinity of the function is at $t = 0$ & 4

→ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

For R.H.L

$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$

$= \lim_{h \to 0} 1 + h^2 + 2h$

Apply limits

$$= 1 + 0^2 + (2(0)$$

$$= 1$$

For L.H.L

$$\lim_{h \to 0} g(1+h) = 2t+3$$

$$= \lim_{h \to 0} 2(1-h) + 3$$

$$= \lim_{h \to 0} 2 - 2h + 3$$

Apply limit

$$= 2 - 2(0) + 3$$

$$= 5$$

$$R.H.L \neq L.H.L = g(t) = 5$$

$\Rightarrow$ Now at $t = 4$

$$g(4) = 2(4) + 3$$

$$= 8 + 3$$

$$= 11$$

For R.H.L

$$\lim_{h \to 0} g(1+h) = \lim_{h \to 0} 2(1+h) + 3$$

$$\lim_{h \to 0} 2 + 2h + 3$$

Apply limits

$$= 2 + 2(0) + 3 \Rightarrow 5$$

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For L.H.L

$$\lim_{h\to 0} g(1-h) = 12$$

$g(4) = R.H.L \neq L.H.L$

point of discontinity is at $t=4$

(b) Find, if they exist

i) $\lim_{t\to 3} g$

For $g(t) = t^2$

R.H.L

$$\lim_{h\to 3} g(1+h) = \lim_{h\to 3} (1+h)^2$$

$= \lim 1 + h^2 + 2h$

Apply limits

$= 1 + 3^2 + 2(3) \Rightarrow 16$

L.H.L

$$\lim g(1-h) = \lim_{h\to 3} 2t + 3$$

$= \lim_{h\to 3} 2(1-h) + 3$

$= \lim_{h\to 3} 2 - 2h + 3$

Apply limits

$= 2 - 2(3) + 3$

$= 2 - 6 + 3$

$= -1$

$R.H.L \neq L.H.L$

(does not exist Since L.H.L is -ive)

Question No 2

Find the maclaurin's Series for

$y(x) = x^2 + \sin x$.

ANS:-

Solution:

$y(x) = x^2 + \sin x$

Since we know that the maclaurin's Series is

$$y(x) = y(x_0) + y'(x_0)(x - x_0) + \frac{y''(x_0)(x - x_0)^2}{2!} + \text{-------} +$$

put $x_0 = 0$

$$y(x) = y(0) + (x-0)\, y'(0) + \frac{(x-0)^2\, y''(0)}{2!} + \text{------} +$$

$$y(x) = y(0) + x\,y'(0) + \frac{x^2\, y''(0)}{2!} + \text{------} + \rightarrow (1)$$

Now find

$y(0) = ?$

$y(1) = x^2 + \sin x$

$y(0) = 0 + \sin 0$

$y(0) = 0 + 0$

$\boxed{y(0) = 0}$

$y(x) = x^2 + \sin x$

$\frac{d}{dx} y(x) = \frac{d}{dx} x^2 + \frac{d}{dx} \sin x$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos(0)$

$= 0 + 1$

$\boxed{y'(0) = 01}$

Since;

$y'(x) = 2x + \cos x$

$d/dx\, y'(x) = 2\frac{d}{dx} x + \frac{d}{dx} \cos x$

$y''(x) = 2 - \sin x$

$y''(0) = 2 - \sin 0$

$= 2 - 0$

$y''(0) = 2$

Now

$y''(x) = 2 - \sin x$

$\frac{d}{dx} y''(x) = \frac{d}{dx} 2 - \frac{d}{dx} \sin x$

$= 0 - \cos x$

$\because \cos 0 = 1$

$= 0 - \cos x$

$y''(0) = - \cos 0$

$\boxed{y''(0) = -1}$

put equ (1)

$$Y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \text{------} +$$

$$= x + \frac{\cancel{2}x^2}{\cancel{2}!} - \frac{x^3}{3!} + \text{------} +$$

$$= x + x^2 - \frac{x^3}{3!} + \text{------} +$$

$\cancel{= x}$

So

$$Y(x) = x + x^2 - \frac{x^3}{3!} + \text{------} +$$

Question NO 3

part 1) Find y'' given

$$1+xy = x^2+y^2$$

ANSWER:-

Solution:-

$$1+xy = x^2+y^2$$

taking $\frac{d}{dx}$ on both side

$$1+\frac{d}{dx}x\frac{d}{dx}y = \frac{d}{dx}x^2+\frac{d}{dx}y^2$$

$$\Rightarrow 1+(1)\frac{dy}{dx} = 2x+2y\frac{dy}{dx}$$

$$1+\frac{dy}{dx} = 2x+2y\frac{dy}{dx}$$

$$\frac{dy}{dx}-2y\frac{dy}{dx} = 2x-1$$

$$\frac{dy}{dx}(1-2y) = 2x-1$$

$$\Rightarrow \boxed{\cancel{\frac{dy}{dx}\cdot(1-2y)}}$$

$$\frac{dy}{dx} = \frac{2x-1}{1-2y} \qquad \boxed{y' = \frac{2x-1}{1-2y}} \rightarrow 1$$

differntiate again 1

$\frac{d}{dx} y' = \frac{d}{dx}\left(\frac{2x-1}{1-2y}\right)$ page = 09

$$y'' = \frac{(1-2y)\frac{d}{dx}(2x-1) - (2x+1)\frac{d}{dx}(1-2y)}{(1-2y)^2}$$ use Quoitent Rule

$$= \frac{(1-2y)(2) - (2x-1)(-2y')}{(1-2y)^2}$$

$$= \frac{(2-4y) - (2x-1)(-2y')}{(1-2y)^2}$$

from eq (1)

$$y' = \frac{2x-1}{1-2y}$$ put in above

$$y'' = \frac{(2-4y) - (2x-1) - 2\left(\frac{2x-1}{1-2y}\right)}{(1-2y)^2}$$

$$= \frac{2(1-\cancel{2y})}{(1-2y)^{\cancel{2}}} - \frac{(2x-1)(2x-1)(-2)}{(1-2y)^3}$$

$$\Rightarrow y'' = \frac{2}{1-2y} - \frac{-2(2x-1)^2}{(1-2y)^3}$$

ANS

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part 2) Find y' by using logrithmic differentation

$$y = x^3(1+x)^9 e^{6x}$$

ANSWER:-

Sol:- $y = x^3(1+x)^9 e^{6x}$

"Take ln on both Sides".

$\Rightarrow \ln(y) = \ln\left(x^3(1+x)^9 e^{6x}\right)$

$\frac{d\ln y}{dx}$ $\Rightarrow \ln(x^3(1+x)^9 + \ln e^{(6x)}$

d

$\Rightarrow \ln(x^3(1+x)^9 + \ln e^{(6x)}$

$\Rightarrow \ln x^3 + \ln(1+x)^9 + 6x$

$\Rightarrow 3\ln x + 9\ln(1+x) + 6x$

NOW;

$$\frac{d\ln(y)}{dx} = \frac{d}{dx}\left(3\ln x + 9\ln(1+x) + 6x\right)$$

$$\Rightarrow 3\frac{d}{dx}\ln x + 9\frac{d}{dx}\ln(1+x) + 6\frac{dx}{dx}$$

$$\Rightarrow 3\cdot\frac{1}{x} + 9\cdot\frac{1}{1+x} + 6$$

$$\frac{d\ln(y)}{dx} = 3/x + \frac{9}{x+1} + 6$$

THE END