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Page #01:-

QNO 1:-

Find the Solution of the Following.

1(a)

The Sum of Two number is K Find the minimum value of the Sum Cubes.

ANS:-

Solution:

Let

x and y = the numbers.

Z = Sum of their Cubes.

$$K = x + y$$

$$y = K - x \quad \cdots \quad Z = x^3 + y^3$$

$$Z = x^3 + (K - x)^3$$

$$\frac{dZ}{dx} = 3x^2 + 3(K-x)^2(-1) = 0$$

$$x^2 - (K^2 - 2Kx + x^2) = 0$$

$$x = 1/2K$$

$$y = K - \frac{1}{2K}$$

$$y = \frac{1}{2K} \qquad Z = \left(\frac{1}{2K}\right)^3 + \left(\frac{1}{2K}\right)^3$$

$$Z = \frac{1}{4K^3}$$

QNO (b):- The Sum of Two positive number is 2 find the Smallest value possible for the Sum of the Cube of one number and the Square of the other.

ANS:-

Solution:

Let x and y = the numbers

$x + y = 2 \longrightarrow$ eq (1)

$1 + y' = 0 \longrightarrow$ eq (2)

$$\frac{dz}{dx} = 3x^2 + 2yy' = 0$$

$$3x^2 + 2y(-1) = 0$$

$$y = 3/2x^2$$

from equation (1)

$$x + 3/2x^2 = 2$$

$$2x + 3x^2 = 4$$

$x = 0.8685$ and -1.5352

Use

$x = 0.8685$ ~~and~~

$y = 32(0.86852)$

$y = 1.1315$

$Z = 0.86853 + 1.13152$

$$\boxed{Z = 1.9354}$$ Answer.

QNO(02):-

Let $f(x)$ -- differential function such that $f(3) = 12$, $f'(3) = 2$. Estimate the value of $f(3.5)$ using the local approximation at $a = 3$.

Solution:-

$$f(x) \approx L(x)$$

$$= f(a) + f'(a)(x - a)$$

We just need to plug in the known values and Calculate value of $f(3.5)$

$$L(x) = f(3) + f'(3)(x - 3)$$

$$= 12 - 2(x - 3) = 18 - 2x$$

Then $f(3.5) \approx 18 - 2 \cdot 3.5 = 11$ Ans.

(2)

QNO 2:- (b)

Estimate $3\sqrt{9}$ using a linear approximation at $a = 8$.

ANS:- Solution

Let $f(x) = \sqrt[3]{x}$

We want to Estimate

$f(8) = 3\sqrt{9}$

$f(a) = f(8) = 3\sqrt{9}$

For the linear approximation we'll need $f'(8)$ as well

$L(x) = x \sqrt[3]{x-5} x+1$

at $a = 8$

$f(8) = 8 - 5\sqrt{8+1} \Rightarrow 3\sqrt{9}$

$$\boxed{f(8) = 3\sqrt{9}}$$

QNO3:- Solve the following differential equation.

$$2xy - 9x^2 + (2y + x^2 + 1)\, dy/dx = 0$$

ANS:- Solution:-

$$(2xy - 9x^2)dx + (2y + x^2 + 1)dy = 0$$

$$P(x, y) = (2xy - 9x^2) \text{ ---------}$$

$$Q(x, y) = 2y + x^2 + 1$$

$$\frac{dP}{dy} = 2 = \frac{dQ}{dx}$$

$\therefore U(x, y)$ exists where

$$du = \frac{du}{dx}dx + \frac{du}{dy}dy$$

$$= P\,dx + Q\,dy = 0$$

Giving (i) $\frac{du}{dx} = 2xy - 9x^2$

(ii) $\frac{du}{dy} = 2y + x^2 + 1$

Integrate i): $U = \frac{1}{2}x^2(y - 9x)$

Differentiate:-

$$\frac{dU}{dy} = \frac{1}{2}x^2y - \frac{9}{2}x^3 \quad \text{using (ii)}$$

$$\therefore \frac{d\phi}{dy} = \frac{1}{2}x^2(y - 9x)$$

$$\boxed{\phi y = \frac{1}{2}x^2y - \frac{9}{2}x^3}$$