Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: Sir pir mehar ali shah **Total Marks:** 30

Student Details

Name: Talha Khan **Student ID:** 13845

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1

Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $x[n] = \{ 2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	Marks 5 CLO 2
	(b)	Compute the convolution y(n) of the following signal $x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$ $h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{3})^{n}, & n \ge 0 \\ (\frac{1}{4})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

(1)

Name: Talha Khan ID = 13845

Q1(a) $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

(i) <u>Sol</u>:- minimum sampling rate
Nyquist criteria:

$$f_s \geq 2f_{max} \qquad \because f = \frac{w}{2\pi}$$

$$f_1 = \frac{100\pi}{2\pi} \qquad f_2 = \frac{200\pi}{2\pi}$$

$$f_1 = 50\ Hz \qquad f_2 = 100\ Hz$$

So

f_2 is max (greater than f_1)

$f_s \geq 2 \times 100\ Hz$

Sampling frequency to avoid aliasing.

Q(a)

Part(ii) $F_s = 100\,Hz$.

<u>Sol</u>:- we have

$F_s = 100\ Hz$

So,

f_1 becomes

$f_1' = f_1 / F_s$

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$$f_1' = \frac{50}{100} = 0.5\ Hz$$

f_2 becomes

$$f_2' = \frac{f_2}{100} = \frac{100}{100}$$

$$f_2' = 1\ Hz$$

So $w_1' = 2\pi f_1'$ $\quad$ $w_2' = 2\pi f_2'$

$w_1' = 2\pi \times 0.5$ $\quad$ $w_2' = 2\pi \times 1$

$\boxed{w_1' = \pi}$ $\quad$ $\boxed{w_2' = 2\pi}$

$$x[n] = 3\cos 100\pi n + 4\sin 200\pi n.$$

The signal becomes

$$x[n] = 3\cos \pi n + 4\sin 2\pi n$$

* Effect :- The effect of this sampling rate on the newly generated discrete time signal is that, there will be not present unwanted components in the reconstruction of the signal and we can reconstruct the original signal.

Name: Talha Khan (3) ID: 13845

(a) (iii) Sol:- $x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The folding frequency of the sampled signal is:

$$\text{folding frequency} = \frac{F_s}{2} \Rightarrow \frac{100}{2}$$

$$= 50\ Hz.$$

we have frequency of the origional signal.

$$f_1 = 50\ Hz \quad , f_2 = 100\ Hz$$

So both the frequency are either equal or greater the folding frequency.

Hence for ideal interpolation we can construct the origional signal

$$\boxed{x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t}$$

The original signal is constructed because we use sampling frequency at Nyquist rate.

We can also reconstruct the signal for sampling frequency above the Nyquist rate.

Talha Khan : 13845

Q1 (b) $x(n) = \begin{cases} 0.5^{-n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$

This is signal is sampled at the rate $F_s = 2\,Hz$

(i) Draw the sampled signal:-

$F_s = 2\,Hz$

$F_s = \frac{1}{T} \Rightarrow T = \frac{1}{F_s} = \frac{1}{2}$

$T = 0.5$ sec

n	$x[n]$
0	$0.5^{-0} = 1$
1	$0.5^{-1} = 0.5$
2	$0.5^{-2} = 0.25$
3	$0.5^{3} = 0.125$
4	$0.5^{-4} = 0.0625$
5	$0.5^{-5} = 0.03125$
6	$0.5^{-6} = 0.015625$
7	$0.5^{-7} = 7.8125 \times 10^{-3}$
8	$0.5^{-8} = 3.90625 \times 10^{-3}$
9	$0.5^{-9} = 1.953125 \times 10^{-3}$

(5)

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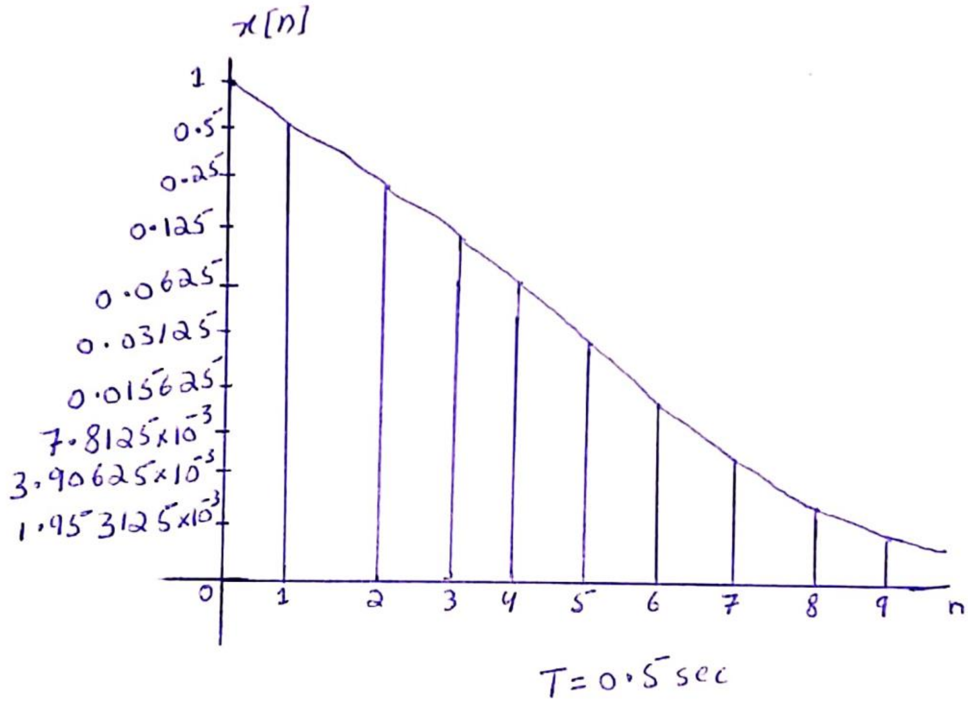

Q1 (b)

(ii) Sol:- $L = 2^n$

$$n = \text{bits} = 3$$

$$L = 2^3 = 8 \text{ Levels}$$

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8} = \frac{1}{8} = 0.125$$

(6)

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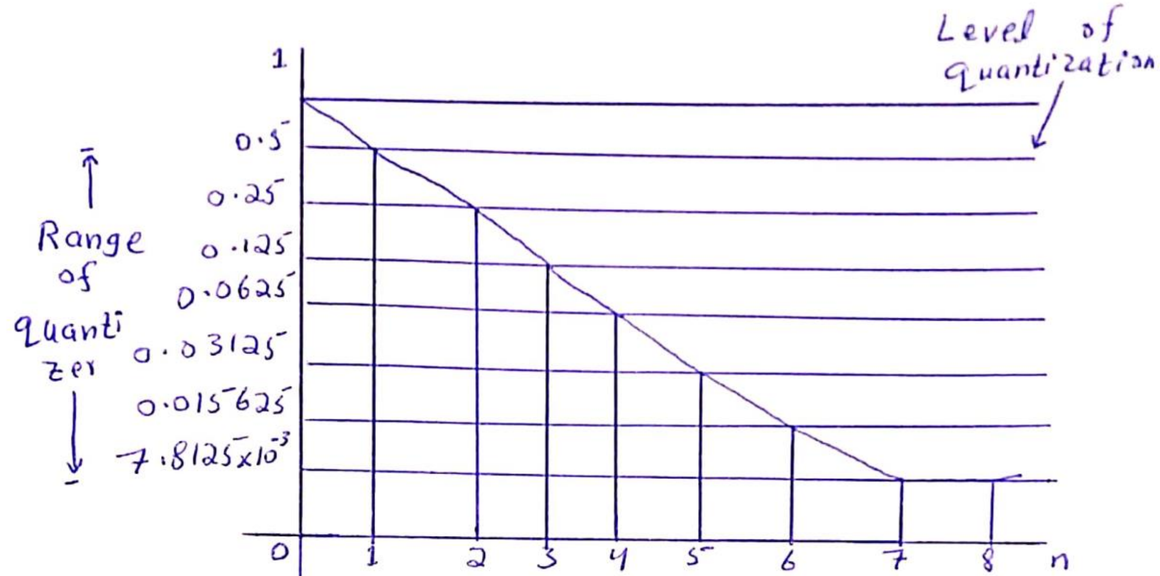

(b) (iii)

$x[n] = (0.5)^n \quad n \geq 0$

n	DF-signal $x[n]$	Truncation $x_q[n]$	Rounding $x_q[n]$	$e_q[n] = x_q[n] - x[n]$
0	1	1	1	0
1	0.5	0.5	0.5	0
2	0.25	0.2	0.3	-0.05
3	0.125	0.1	0.1	0.025
4	0.0625	0.06	0.06	2.5×10^{-3}
5	0.03125	0.03	0.03	1.25×10^{-3}
6	0.015625	0.01	0.02	-4.375×10^{-3}
7	7.8125×10^{-3}	0.007	0.008	-1.875×10^{-4}
8	3.90625×10^{-3}	0.003	0.004	-9.375×10^{-5}
9	1.953125×10^{-3}	0.001	0.002	-4.8875×10^{-5}
10	9.765625×10^{-4}	0.0009	0.001	-2.34375×10^{-5}

(7)

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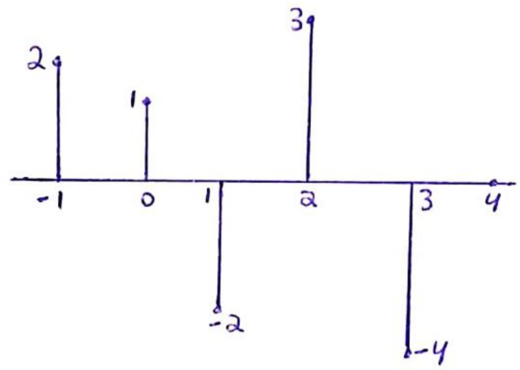
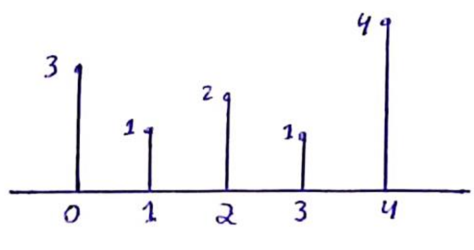
Q2. (a) $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$

$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

Sol:- $x(n) = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$



$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$



(8)

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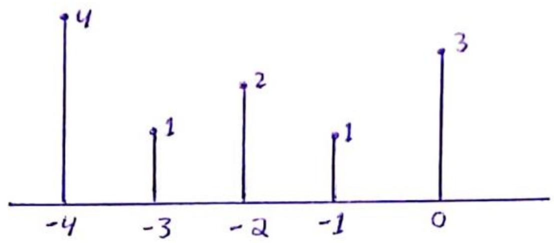
$h[-n] = \{4, 1, 2, 1, \underset{\uparrow}{3}\}$



$y[-2] = 0$

$y[-1] = 6$

$y[0] = 5$

$y[1] = 3\times-2 + 1\times1 + 2\times2 = -6+1+4 = -1$

$y[2] = 3\times3 + 1\times-2 + 2\times1 + 1\times2 = 9-2+2+2 = 11$

$y[3] = 3\times-4 + 1\times3 + 2\times-2 + 1\times1 + 4\times2 = -12+3-4+1+8 = -4$

$y[4] = 1\times-4 + 2\times3 + 1\times-2 + 4\times1 = -4+6-2+4 = 4$

$y[5] = 2\times-4 + 1\times3 + 4\times-2 = -8+3-8 = -13$

$y[6] = 1\times-4 + 4\times3 = -4+12 = 8$

$y[7] = 4\times-4 = -16$

(9)

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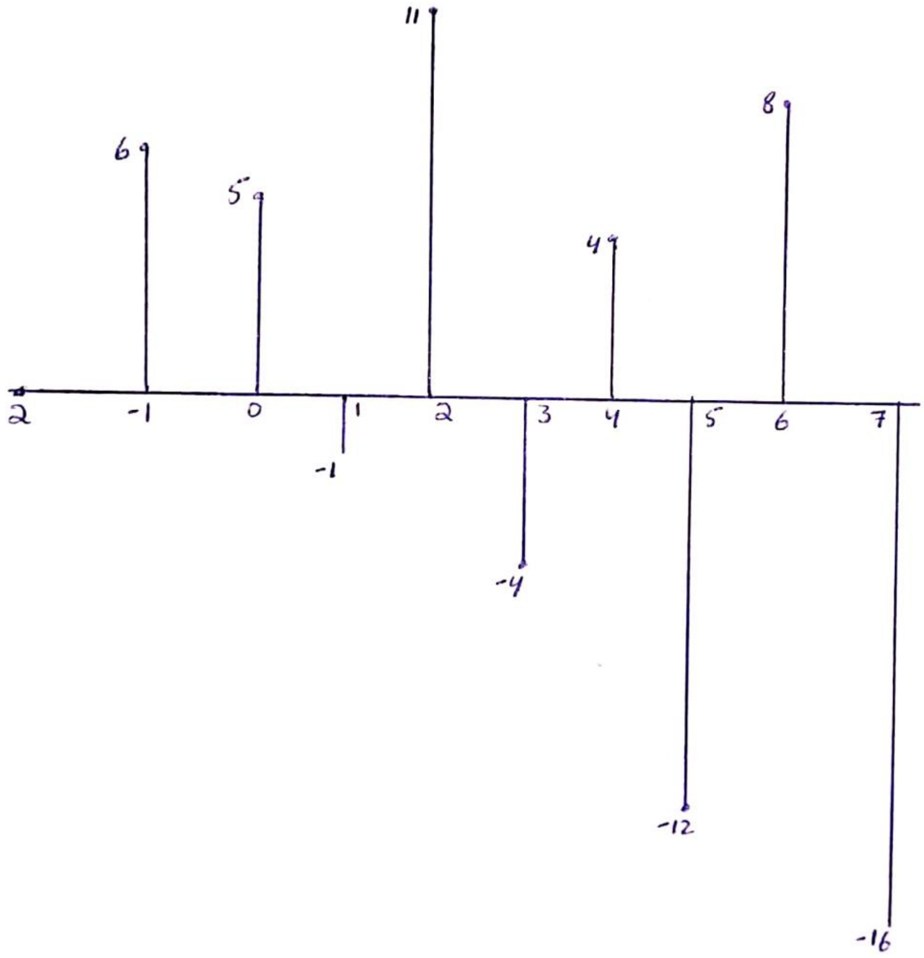
$y[n] = \{0, 6, \underset{\uparrow}{5}, -1, 11, -4, 4, -12, 8, -16\}$



(10)

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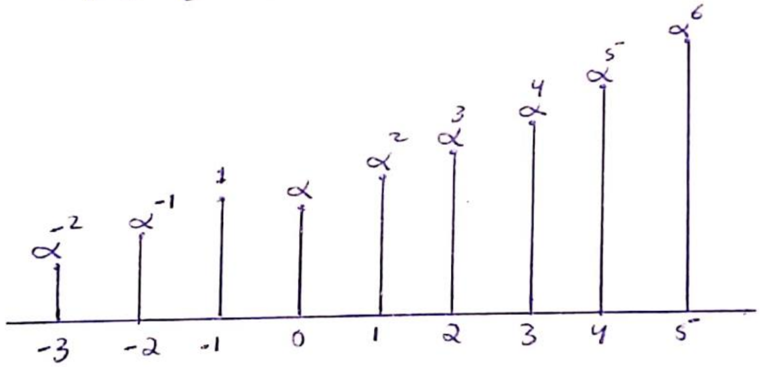
Q2. (b)

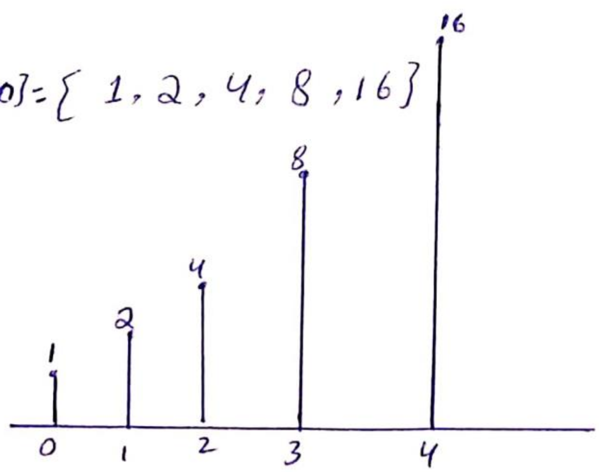
Sol:- $x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$

$h(n) = \begin{cases} 2^{n}, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

$x[n] = \{ \alpha^{-2}, \gamma^{-1}, 1, \alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6} \}$



$h[n] = \{ 1, 2, 4, 8, 16 \}$



(11)

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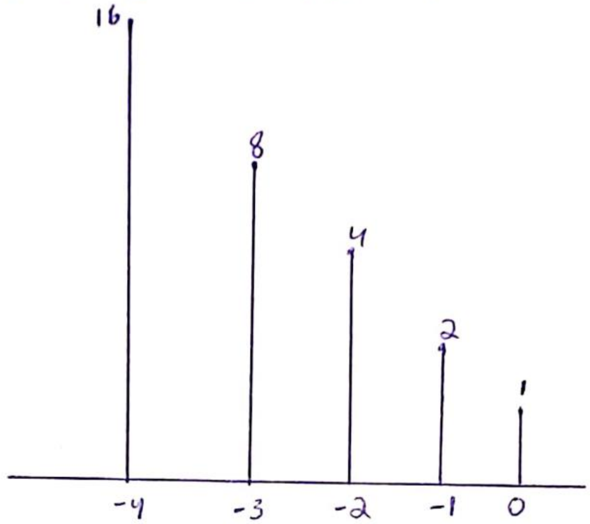
$h[-n] = \{1, 2, 4, 8, 16\}$



$y[-3] = 1\times\alpha^{2}$

$y[-2] = 1\times\alpha^{-1} + 2\times\alpha^{-2}$

$y[-1] = 1\times1 + 2\times\alpha^{-1} + 4\times\alpha^{-2}$

$y[0] = 1\times\alpha + 2\times1 + 4\times\alpha^{-1} + 8\times\alpha^{-2}$

$y[1] = 1\times\alpha^{2} + 2\times\alpha^{1} + 4\times1 + 8\times\alpha^{-1} + 16\times\alpha^{-2}$

$y[2] = 1\times\alpha^{3} + 2\times\alpha^{2} + 4\times\alpha^{1} + 8\times1 + 16\times\alpha^{-1}$

$y[3] = 1\times\alpha^{4} + 2\times\alpha^{3} + 4\times\alpha^{2} + 8\times\alpha^{1} + 16\times1$

$y[4] = 1\times\alpha^{5} + 2\times\alpha^{4} + 4\times\alpha^{3} + 8\times\alpha^{2} + 16\times\alpha^{1}$

$y[5] = 1\times\alpha^{6} + 2\times\alpha^{5} + 4\times\alpha^{4} + 8\times\alpha^{3} + 16\times\alpha^{2}$

$y[6] = 2\times\alpha^{6} + 4\times\alpha^{5} + 8\times\alpha^{4} + 16\times\alpha^{3}$

(12)

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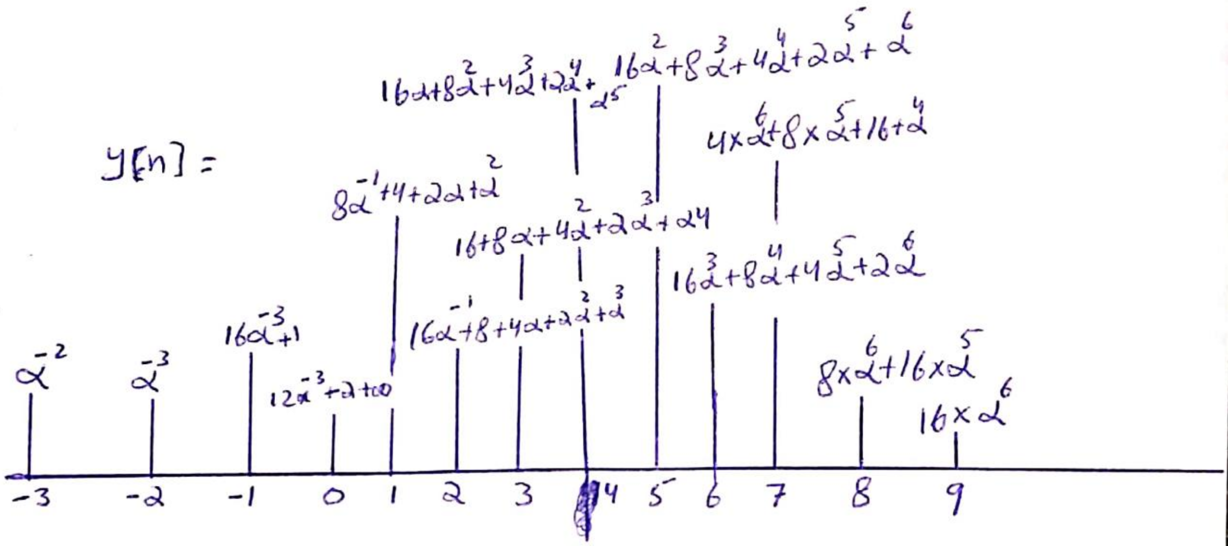
$$y[7] = 4 \times \alpha^6 + 8 \times \alpha^5 + 16 \times \alpha^4$$

$$y[8] = 8 \times \alpha^6 + 16 \times \alpha^5$$

$$y[9] = 16 \times \alpha^6$$

$$y[10] = 0$$

$y[n] =$



Talhakhan (13) 13845

Q3.(i) Sol:- $x[n] = \begin{cases} (1/4)^n, & n \geq 0 \\ (1/3)^{-n}, & n < 0 \end{cases}$

writing in the form of z-transform

$$x(z) = \sum_{n=0}^{\infty} \left(1/4\right)^n z^{-n} + \sum_{n=-\infty}^{\infty} \left(1/3\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-1/3} - 1$$

$$= \frac{1 - 1/_{1/4}\, z^{-1} + 1-\frac{1}{1}-1}{\left(1-1/4\, z^{-1}\right)\left(1-1/3\, z^{-1}\right)} - 1$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1 - 1/3\, z + 1 - 1/4\, z^{-1} - \left(1 - 1/3\, z - \frac{1}{4}z^{-1} + \frac{1}{12}z^{-1}\cdot z\right.}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

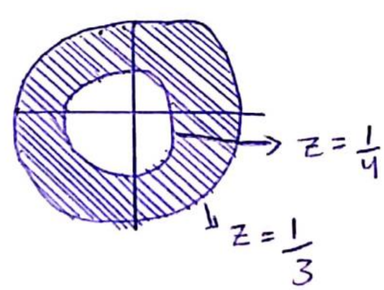
Talha Khan 13845

$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-1+\frac{1}{3}z+\frac{1}{4}z^{-1}+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1+\frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{\frac{13}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

Hence the ROC is $\frac{1}{4} < |z| < 3$.



(15)

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Q 3.

(ii) $x[n] = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

<u>Sol</u>:-

$$x[Z] = \sum_{n=-\infty}^{\infty} \left\{ (1/2)^n - (3)^n \right\} z^{-n}$$

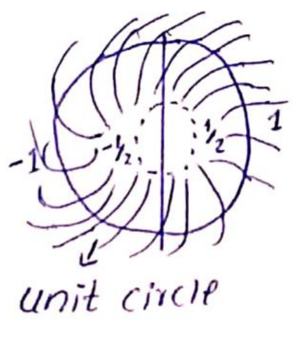
$$x[Z] = \sum_{n=-\infty}^{\infty} (1/2)^n z^{-n} - \sum_{n=-\infty}^{\infty} (3)^n z^{-n}$$

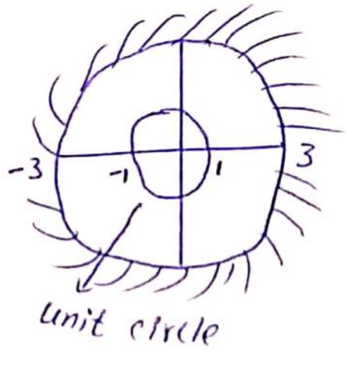
$$x[Z] = \sum_{n=0}^{\infty} (1/2\, z^{-1})^n - \sum_{n=0}^{\infty} (3 z^{-1})^n$$

$$x[Z] = \frac{1}{1 - 1/2\, z^{-1}} - \frac{1}{1 - 3 z^{-1}}$$

$$x[Z] = \frac{z}{z - 1/2} - \frac{z}{z - 3}$$

$|z| > \frac{1}{2}$ $\qquad$ $|z| > 3$





(16)

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