Q1 Let $Z = \dfrac{3x^3 - 5x^2 + 5}{(x^2+1)}$

$$\boxed{\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{d}{dx}f - f\frac{d}{dx}g}{g^2}}$$

Taking diff w.r.t x

$$\frac{dZ}{dx} = \frac{d}{dx}\left[\frac{3x^3 - 5x^2 + 5}{(x^2+1)}\right]$$

$$= \frac{(x^2+1)\frac{d}{dx}[3x^3 - 5x^2 + 5] - (3x^3 - 5x^2 + 5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)(9x^2 - 10x + 0) - (3x^3 - 5x^2 + 5)(2x + 0)}{(x^2+1)^2}$$

$$= \frac{9x^4 - 10x^3 + 9x^2 - 10x - 6x^4 + 10x^3 - 10x}{(x^2+1)^2}$$

$$= \frac{3x^4 + 9x^2 - 20x}{(x^2+1)^2}$$

Ans

Let $z = \frac{(x^2+1)^2}{(x^2-1)}$

Taking diff. w.r.t x

$$\frac{dz}{dx} = \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\cdot 2(x^2+1)\cdot 2x - (x^2+1)^2\cdot(2x)}{(x^2-1)^2}$$

$$= \frac{2x(x^2-1)(x^2+1) - (x^2+1)^2 2x}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)\left[2(x^2-1) - (x^2+1)\right]}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(2x^2-2-x^2-1)}{(x^2-1)^2}$$

$$= \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

Q2(i) $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$

Taking diff. w.r.t x.

$$\frac{dy}{dx} = \frac{d}{dx}\left[(1+2\sqrt{x})^3 x^{2/3}\right]$$

$$\boxed{\frac{d}{dx}(f \cdot g) = f\frac{d}{dx}g + g\frac{df}{dx}}$$

$$= (1+2\sqrt{x})^3 \frac{d}{dx}x^{2/3} + x^{2/3}\frac{d}{dx}(1+2x^{1/2})^3$$

$$= (1+2x^{1/2})^3 \frac{2}{3}x^{2/3-1} + x^{2/3} \cdot 3(1+2x^{1/2})^{3-1} \frac{2}{2} \cdot 1 \cdot x^{1/2-1}$$

$$= (1+2x^{1/2})^3 \frac{2}{3}x^{-1/3} + x^{2/3}x^{-1/2} \cdot 3(1+2x^{1/2})^2$$

$$= \frac{2}{3}x^{-1/3}(1+2x^{1/2})^3 + 3x^{1/6}(1+2x^{1/2})^2$$

b

$$y = \sqrt{\frac{1-x}{1+x}}$$

$$y = (1-x)^{1/2}(1+x)^{-1/2}$$

Taking diff. w.r.t x

$$\frac{dy}{dx} = (1-x)^{1/2}\frac{d}{dx}(1+x)^{-1/2} + (1+x)^{-1/2}\frac{d}{dx}(1-x)^{1/2}$$

$$= (1-x)^{1/2}\left(-\frac{1}{2}\right)(1+x)^{-1/2-1}(0+1) + (1+x)^{-1/2}\frac{1}{2}(1-x)^{\frac{1}{2}-1}(0-1)$$

$$= -\frac{1}{2}(1-x)^{1/2}(1+x)^{-3/2} + (1+x)^{-1/2}\left(-\frac{1}{2}\right)(1-x)^{-1/2}$$

$$= -\frac{1}{2}(1+x)^{-1/2}(1-x)^{1/2}\left[\frac{1}{1+x} + \frac{1}{(1-x)^{1/2}}\right]$$

$$= -\frac{1}{2}\left[\frac{1-x}{1+x}\right]^{1/2}\left[\frac{1}{1+x} + \frac{1}{(1-x)^{1/2}}\right]$$

Q3 (g)

$$\int \frac{1}{x^{3/2}}\,dx$$

$$\int x^{-3/2}\,dx$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C}$$

$$\int x^{-3/2}\,dx = \frac{x^{-3/2+1}}{-\frac{3}{2}+1} + C$$

$$= -2\,x^{-1/2} + C$$

$$\int \frac{1}{x^{3/2}}\,dx = -\frac{2}{\sqrt{x}} + C$$

b

$$\int \frac{1}{(6x+7)^6}\,dx$$

Let $z = 6x+7$

$dz = 6\,dx$

$dx = \frac{dz}{6}$

Now $\int \frac{1}{(6x+7)^6}\,dx = \int \frac{1}{z^6}\,\frac{dz}{6}$

$= \frac{1}{6}\int z^{-6}\,dz = \frac{1}{6}\,\frac{z^{-6+1}}{-6+1} + C$

$= \frac{1}{6}\,\frac{z^{-5}}{(-5)} + C$

$= -\frac{1}{30}(6x+7)^{-5} + C$

$= \frac{-1}{30(6x+7)^5} + C$