Name :- Ihtisham fida

I.D :- 7209

Submitted To :- "Shomaila Mazhar"

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Contact :- 0345-5329829

Assignment

Differential Eq.

Q.No.1 **(10)**

Solve the following objective type questions.

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~~i.~~ The order of matrix A is m × p and the order of B is p × n. Then the order of matrix AB is?

~~ii.~~ The number of non-zero rows in an Echelon form?

~~iii.~~ If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then a=?

~~iv.~~ If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

~~v.~~ The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

~~vi.~~ Solution of $\frac{dy}{dx} + 2xy = y$?

~~vii.~~ The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

~~viii.~~ The order and degree of differential equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is?

~~ix.~~ The differential equation $2\frac{dy}{dx} + x^2y = 2x + 3, y(0) = 5$ is?

~~x.~~ $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

Q.No.2 **(10)**

i. Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

ii. Find the Eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Q.No.3 **(10)**

The rate of change in the form of differential equation is given by

$(x^2 + 3y^2)dx - 2xydy = 0$. Find the general solution at x=2 and y=6.

QNO 1 ⇒

i) $A = [a_{ij}]_{m \times p}$ and $B = [b_{ij}]_{p \times n}$

Then the product AB is defined to be the matrix $[C_{ij}]_{m \times n}$ where

$$C_{ij} = [a_{i1}, a_{i2} \ldots\ldots a_{in}] \begin{bmatrix} b_{1j} \\ b_{2j} \\ b_{nj} \end{bmatrix} = a_{i1} b_{1j}, a_{i2} b_{2j} + a_{in} + b_{nj}$$

i-e ⇒ $A.B = [C_{ij}]_{m \times n}$

x ——————— x

(ii) The Number of non-zero raw in an Echeion form of a Matrix determine the rank of the matrix.

x ——————— x

iii) $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix} \Rightarrow |B| = 0$ —— (1)

$\Rightarrow |B| = (1)(a) - (2)(4)$

$\Rightarrow |B| = a - 8$

From (1)

$\Rightarrow a - 8 = 0$

$\boxed{a = 8}$

x ———————————— x

iv) $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix} \Rightarrow |A| = (2i)(-i) - (i)(i)$

$= -2i^2 - i^2$

$\Rightarrow -2(-1) - (-1)$

$\Rightarrow 2 + 1$

$|A| = 3$

x ———————————— x

v) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scaler Matrix.

x ———————————— x

vi) $dy/dx + 2xy = y$.

$\Rightarrow dy/dx = y - 2xy$

$\Rightarrow dy/dx = (1-2x)\,dx$

$\Rightarrow dy/y = (1-2x)\,dx$

$\int dy/y = \int (1-2x)\,dx$

$\Rightarrow \ln y = x - 2x^2/2 + C_1$

$\ln y = (x - x^2) + C_1$

$e^{\ln y} = e^{(x-x^2)+C_1}$

$y = e^{x-x^2} \cdot e^{C_1}$

$\boxed{y = Ce^{x-x^2}}$

vii) $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

Taking Square on Both side

$\Rightarrow \left(\left(\frac{dy}{dx}\right)^3\right)^2 = \left(\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2}\right)^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^6 = 1+\left(\frac{dy}{dx}\right)^2$

$\Rightarrow$ Degree $= 6$

$\Rightarrow$ Order $= 1$

x ———————— x

(5)

viii) The order of $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is

2 But the degree is undefined because

the unknown function "y" is an argument

of transcendental Sin function.

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ Expanding with first Row

$\Rightarrow 1\begin{bmatrix} b & b^2 \\ c & c^2 \end{bmatrix} - a\begin{bmatrix} 1 & b^2 \\ 1 & c^2 \end{bmatrix} + a^2\begin{bmatrix} 1 & b \\ 1 & c \end{bmatrix}$

$\Rightarrow 1\,(bc^2 - cb^2) - a\,(c^2 - b^2) + a^2\,(c-b)$

$\Rightarrow bc\,(c-b) - a\,(c-b)(c+b) + a^2\,(c-b)$

$\Rightarrow (c-b)\left[bc - a\,(c+b) + a^2\right]$

$\Rightarrow (c-b)\left[bc - ac - ab + a^2\right]$

$\Rightarrow (c-b)\;c\,(b-a) - a\,(b-a)$

$\boxed{\Rightarrow (c-b)\;(b-a)\;(c-a)}$ Ans

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ix) $2\frac{dy}{dx} + x^2 y = 2x + 3, \quad y(0) = 5$

Sol:

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2} x^2 y \pm x + 3/2 \quad () \text{ By '2'}$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{2} x (xy - 1) = 3/2$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{2} x^2 y + \frac{1}{2} x + 3/2 \quad \text{integrate}$$

$$\Rightarrow y = \frac{-1}{2} \frac{x^3 y}{3} + \frac{1}{2} \frac{x^2}{2} + \frac{3}{2} x + C_1$$

$$\Rightarrow y = \frac{-1}{6} x^3 y + \frac{1}{4} x^3 + \frac{3}{2} x + C_1 \quad \text{—(A)}$$

Now use condition

at $x = 0, \quad y = 5$

Put in eq — (A)

$$5 = 0 + 0 + 0 + C_1$$

$$\Rightarrow C_1 = 5$$

$$\boxed{y = -\frac{1}{6} x^3 y + \frac{1}{4} x^2 + \frac{3}{2} x + 5}$$

QNO⇒2 ⇒

i)
$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Taking abc comon from each column

$$abc\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Now by Row operation we can get

$$abc\begin{vmatrix} 1 & 1 & 1 \\ a-a & b-a & c-a \\ a^2-a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} \quad \begin{matrix} R_2 - aR_1 \\ R_3 - a^2R_1 \end{matrix}$$

$$abc\begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Now expending by 1st column

$$abc\left[1\begin{vmatrix} b-a & c-a \\ b^2-a^2 & c^2-a^2 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ b^2-a^2 & c^2-a^2 \end{vmatrix} + 0\begin{vmatrix} 1 & 1 \\ b-a & c-a \end{vmatrix}\right]$$

$$abc\left[(b-a)(c^2-a^2)-(c-a)(b^2-a^2)\right]$$

$$abc\left[(b-a)(c-a)(c+a)-(c-a)(b-a)(b+a)\right]$$

$$\boxed{abc\,(b-a)(c-a)(c-b)}$$

(8)

ii) $\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$

Now By Row operation

$$A = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix} \quad R_1 \rightarrow R_2$$

$$A = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 0 & 5 & -3 & -2 \\ 0 & -4 & 4 & 0 \\ 0 & -1 & 1 & 2 \end{vmatrix} \quad \begin{matrix} R_2 + 2R_1 \\ R_3 - R_1 \end{matrix}$$

$$A = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & -4 & 4 & 0 \\ 0 & 5 & -3 & -2 \end{vmatrix}$$

$$A = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & -8 & 8 \end{vmatrix} \quad \begin{matrix} R_3 - 4R_2 \\ R_4 + 5R_2 \end{matrix}$$

$$A = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 0 & 0 \end{vmatrix} \quad R_3 + R_4$$

Now to find the eigen value we will calculate $\det(A - \lambda I)$

$$A - \lambda I = \begin{vmatrix} -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 8 & -8 \\ 0 & 0 & 0 & 0 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} -1-\lambda & 3 & -1 & -1 \\ 0+\lambda & +\lambda & -1 & 2 \\ 0 & 0 & 8-\lambda & -8 \\ 0 & 0 & 0 & 0-\lambda \end{vmatrix}$$

$\det(A - \lambda I) = 0$

$= 1(-1-\lambda)(-1-\lambda)(8-\lambda)(-\lambda) = 0$

$-1-\lambda = 0, \quad 1-\lambda = 0, \quad 8-\lambda = 0, \quad -\lambda = 0,$

$\Rightarrow \lambda = -1, \quad \lambda = -1, \quad \lambda = 8, \quad \lambda = 0$

are the required eigen values of the matrix